Research Article

Taking birth year into account when analysing effects of maternal age on child health and other outcomes: The value of a multilevel-multiprocess model compared to a sibling model

Øystein Kravdal

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Øystein Kravdal[1]

Abstract

BACKGROUND
When analysing effects of maternal age on child outcomes, many researchers estimate sibling models to control for unobserved factors shared between siblings. Some have included birth year in these models, as it is linked to maternal age and may also have independent effects. However, this creates a linear-dependence problem.

OBJECTIVE
One aim is to illustrate how misleading the results may actually be when attempts are made to separate effects of maternal age and birth year in a sibling analysis. Another goal is to present and discuss the multilevel-multiprocess model as an alternative.

METHODS
Infant mortality was chosen as the outcome. Births and infant deaths were simulated from a multilevel-multiprocess model that included two equations for fertility, with a joint random effect, and one equation for infant mortality, with another random effect. The two random effects were set to be correlated. The effects of the independent variables were taken from simpler models estimated from register data. Various sibling models and multilevel-multiprocess models were estimated from these simulated births and deaths. In some simulations, two standard assumptions about the random effects were intentionally violated.

CONTRIBUTIONS
The paper illustrates how problematic it is to include both maternal age and birth year in a sibling model. Also, if only maternal age is included, but along with other reproductive variables, small categories should be used. It is argued that a multilevel-multiprocess model may be used instead to separate effects of maternal age and birth year, but this approach also has limitations, which are discussed.

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[1] Department of Economics, University of Oslo, and Centre for Fertility and Health, Norwegian Institute of Public Health. Email: okravdal@econ.uio.no.

1. Introduction

1.1 Effects of maternal age on child health

There is a large literature on how maternal age is linked to child health, and the pattern is rather complex. On the one hand, studies have shown high child mortality and other adverse health outcomes at a low age among children whose mother was young when they were born (Finlay, Özaltin, and Canning 2011), and long-term disadvantages for these children have also been suggested (Bjørngaard et al. 2015; McGrath et al. 2004). On the other hand, there also seem to be disadvantages for children with old mothers. For example, some studies have shown a particularly high chance of being born pre-term or with low birth weight (Jacobsson, Ladfors, and Milsom 2004), which in turn may have implications for long-term health, education, and labour market outcomes (Black, Devereux, and Salvanes 2007). Furthermore, the chance of chromosomal or congenital abnormalities increases as a woman approaches the end of her reproductive period (Yoon et al. 1996), and advanced maternal age has been linked to childhood cancer (Johnson et al. 2009), diabetes (Cardwell et al. 2009), and more rare conditions such as autism (Lee and Grath 2015) and bipolar disorders (Menezes et al. 2010). A recent study indicates adverse consequences of high maternal age even for adult all-cause mortality (Barclay and Myrskylä 2018).

The relationship between maternal age and offspring health partly reflects genetic mechanisms in the mother, as well as in the father, whose age is typically not very different (Lee and Grath 2015; Menezes et al. 2010; Johnson et al. 2009; Yoon et al. 1996). Other physiological pathways may also contribute. For example, adverse effects of early motherhood may occur partly as a result of a particularly heavy nutritional burden during pregnancy (from feto-maternal competition) and physiological immaturity (Chen et al. 2007; Gibbs et al. 2012), especially in poor settings. It has also been suggested that hormonal factors with implications for the intrauterine environment are involved (Park et al. 2008).

Another possible pathway is so-called "weathering" (Geronimus 1992), which means that higher age increases the chance that the mother suffers from hypertension, diabetes, or other chronic diseases, with possible implications for the child's health. This is particularly relevant in socially disadvantaged populations (Powers 2013; Love et al. 2010; Goisis and Sigle-Rushton 2014). More 'social' mechanisms probably also contribute. For example, younger parents typically stay alive for a longer part of the child's life, and there are more likely to be grandparents available for support as well. Another 'social' mechanism, which operates in the opposite direction and probably is more important, is that parents' economic situation typically improves as their age increases, and they may also accumulate more knowledge as they become older

(Powell, Steelman, and Carini 2006). This resource advantage associated with late motherhood may contribute to the better cognitive development and academic achievements observed in some studies among children with relatively old mothers (Tearne 2015), and it may also influence the children's health in the long term for other reasons.

1.2 Controlling for selection through a sibling fixed-effects analysis

In addition to being a result of mechanisms such as those just mentioned, statistical relationships between maternal (or paternal) age at birth and child health (or other child outcomes) reflect joint determinants. These include the parents' economic resources, work situation, education (operating not only through economic factors), health, and lifestyle preferences, whether the grandparents are able to provide various types of support, and resources and political attitudes in the community (which have implications, for example, for access to day-care centres and health services). For most purposes, one would like to control for these joint determinants as well as possible. Some of them may be quite adequately measured in the available data, and can then be included in a regression model for child health. However, realistically there will always be many unmeasured (or in practice unmeasurable) confounders. Stated differently, other child health determinants exist apart from maternal age and other characteristics that are measured and included in the child health model, and these ignored determinants may be correlated with maternal age.

In a number of studies, this problem related to unmeasured potential confounders has been partly solved by estimating sibling models, which means that the health among siblings is compared. Such an approach at least controls for maternal, household, and environmental factors that are shared between siblings. In principle, the importance of parental age for child health may also be analysed by means of 'instruments', i.e., factors (such as certain family events or policy changes) that can reasonably be assumed to lead to a lower or higher maternal age at birth, while not otherwise having an influence on child health. However, it is often difficult to find factors satisfying this requirement.

To illustrate intuitively how a sibling analysis works, consider a population that includes one mother who had a child at age 25 and another child at age 30, and another mother who had one child at age 23 and another child at age 26. The estimation of the effect of maternal age on child health essentially involves comparison of the differences in health between the first mother's two children and between the second mother's two children with the corresponding maternal age differences – along with similar differences among other mothers' children.

There may be differences between siblings that are linked to, but not a result of, the differences in maternal age, and which therefore would be particularly interesting to control for. For example, there is an obvious association between maternal age and birth order: The sibling who was born when the mother was older is bound to have a higher birth order. If we want an estimate that we can use to predict the implications of postponing the birth of the next child, or the first child, we would need to control for that. Furthermore, the siblings may have been born after different birth intervals, and one may want to separate the effect of birth interval from those of maternal age and birth order.

1.3 The need to control for birth year, which is impossible in a sibling analysis

Another important relationship is that the sibling born when the mother was oldest necessarily was also born in a later calendar year. Thus, in the absence of a control for birth year, the estimated effect of maternal age will reflect the sum of the effect of birth year and a more or less 'pure' effect of maternal age (depending on how many other factors linked to maternal age and affecting child health that are controlled for). Birth year may have an independent effect on child health because of, for example, general improvements in maternal nutrition (in poor settings), pre-natal care, or medical treatment after birth. If the intention is to predict the implication of postponing the birth of the first or the next child, which is bound to result in a correspondingly later year of birth, knowledge about such a combined effect (based on this kind of analysis that at least controls for unobserved factors shared between siblings) may be sufficient. However, it could also be argued that it would be valuable to know the separate effects of maternal age and birth year and then base predictions on the sum of these. This is because, while the maternal age effect may well remain quite constant some time into the future (and perhaps especially the component that is produced by biological rather than social mechanisms), there may be more doubt about the persistence of the birth-year effect. There may be good reasons both to assume that the future effect will not be as strong as in the past and to assume the opposite, and by allowing the summation of effects people are free to make their own assumptions. Additionally, obtaining effects that are as 'pure' as possible is typically a goal from a scientific perspective. For example, the aim may be to learn about bio-medical processes linked to maternal age, and then the contribution from the birth-year effect (and as many as possible of the social effects) definitely has to be wiped out.

Similarly, one may want to separate out the birth-year effect when analysing effects of maternal age on various social outcomes (which themselves are important and also may have implications for later health). For example, birth year may affect a

child's educational career because of increasing pressure to take post-secondary education or larger economic support to do so (Breen 2010).

The recent register-based analysis of four social and health effects of maternal age by Barclay and Myrskyla (2016) is one example of an investigation aiming to separate out the birth-year effect. Another is the study by Bjørngaard et al. (2015), who add a crude control for birth year when estimating the importance of maternal age for offspring suicide risks. Kudamatsu (2012) and Molitoris (2018) include both maternal age and birth year in their studies of infant mortality, but without paying much attention to the effects of these two variables, as their focus is on other mortality determinants.

However, it is not possible to separate the effects of maternal age and birth year in a sibling analysis without making some bold assumptions. As explained very well by Keiding and Andersen (2016), this is because the child's birth year (B) minus the mother's age at the child's birth (A) is the mother's birth year (MB), which is the same for all siblings and can be considered part of the sibling fixed effect. Thus, we are faced with a linear-dependence problem often referred to by demographers as an age-period-cohort problem. Regardless of how the effects of A and B are specified – they may, for example, be included as grouped variables – the model is indistinguishable from a model where a linear trend is added to the effect of A and subtracted from the effect of B. Of course, effects can be estimated if assumptions are made about one of the linear components, but typically there is no theoretical basis for making such assumptions.

To illustrate the problem mathematically in a very simple way with linear effects of maternal age and birth year, assume that a certain continuous health outcome Y_{ij} for child i in sibling group j is given by

$$Y_{ij} = b_0 + b_1 A_{ij} + b_2 B_{ij} + e_j + \varepsilon_{ij}, \qquad (1)$$

where e_j (sibling fixed effect) is an addition to Y_{ij} that is shared by the siblings and ε_{ij} is a child-specific error term. In a sibling analysis where there are only two siblings (to further simplify the example), essentially the following equation is estimated, which involves differences in Y, A, and B between the siblings:

$$\Delta Y_j = b_1 \Delta A_j + b_2 \Delta B_j + \Delta \varepsilon_j.$$

Since $\Delta A_j = \Delta B_j$, the right hand side can alternatively be written as $(b_1 + b_2) \Delta A_j$ or $(b_1 + b_2) \Delta B_j$. Thus, either A or B can be included, and in both cases the estimated effect is the sum $b_1 + b_2$ of the true effects: there is no way to find out how large b_1 and b_2 are. The situation is the same if a conditional logit model for a dichotomous outcome variable is estimated. Consequently, it makes good sense to only report the combined effect $(b_1 + b_2)$, and if an additional step is taken by including both A and B in the

models, readers should be warned (as in Barclay and Myrskylä 2018) that there is much uncertainty about the corresponding separate effects.

1.4 Multilevel-multiprocess models as a possible alternative

As should be clear by now, the idea behind the sibling model is to add sibling fixed effects (e_j in Equation 1) to control for unobserved characteristics shared by siblings that may affect their health, and that may also be correlated with maternal age (A in Equation 1), other reproductive factors, or other types of variables included in the model. Is there another way to take into account unobserved determinants of child health that are shared by siblings, and that are somehow linked to the reproductive factors? One possibility might be to explicitly model the link between the unobserved health determinants and the reproductive factors using random effects, and it would make good sense to do that by considering the birth rates as the 'building blocks' behind the reproductive factors. More concretely, a model might be specified that includes equations for parity transitions and child health, and in which there are some observed determinants (including reproductive factors in the health equation) plus mother-specific fertility and health random effects that represent effects of unobserved characteristics of the mother (and her household and the environment where she lives), and that are correlated with each other. A simple version of this would be to include one constant health random effect, reflecting that the health of all siblings is influenced by the unobserved factors in the same way, as well as one constant fertility random effect, reflecting that there is a similar contribution from the unobserved factors for all parity transitions. Such models are often referred to as multilevel-multiprocess models (the highest 'level' being the mother, and the second and lowest level the children or the mother's different parity transitions, while the 'processes' are fertility and child health). It is typically assumed when estimating such models that the random effects are normally distributed and uncorrelated with the observed variables (except that the health random effect in this case obviously would be correlated with the reproductive factors in the health equation, because the latter are realizations of a model including a fertility random effect that is correlated with the health random effect). Multilevel-multiprocess models of this type are central to this paper, but other versions of multilevel-multiprocess models will also receive some attention.

As with a sibling model, a multilevel-multiprocess model controls for constant unobserved characteristics at the mother level or higher that affect both the mother's fertility and her children's health, but it is done differently and does not involve comparison of differences between siblings in the same way. In contrast to the sibling

model, women with only one child also contribute to the estimation, although not in the estimation of the distribution of the random effects.

Identification of a multilevel-multiprocess model of the type specified here requires that there are at least some individuals for whom there are at least two observations or 'spells' in each 'process'. This requirement is obviously satisfied in this case, since many women have more than one child. (See Steele, Sigle-Rushton, and Kravdal 2009, Väisänen 2017, and Kravdal 2018 for examples of how multilevel-multiprocess models have been used in demographic research recently.)

1.5 Aims

One goal of this paper is to illustrate how misleading the results can be when birth year is added in a sibling fixed-effects analysis of effects of maternal age, by means of a simulation experiment where infant mortality (rather than another indicator of child health) is the outcome. Although the problem is obvious mathematically and well known to many researchers, it may be useful to see some examples of its importance in practice. The reason for using a simulation-based approach is that if the simulated population satisfies the assumptions of the model and large or many replications are made, a good estimation procedure should produce effect estimates close to the corresponding effects used in the simulation (the 'true' effects). The other goal, also involving simulation, is to discuss the value of a multilevel-multiprocess model as an alternative to the sibling model.

More specifically, the first step of the analysis (presented in Section 2) is to simulate populations with characteristics or behaviours that are handled well by the sibling model, i.e., constant unobserved mortality determinants somehow link to the reproductive factors. This is done by simulating from a multiprocess-multilevel model, just as described in the preceding sub-section. A link is established between the unobserved mortality determinants and the reproductive factors in the mortality equation by drawing constant mother-specific contributions to fertility and mortality (random effects) from a bivariate normal distribution with a certain correlation, chosen arbitrarily. The reproductive factors are birth order and birth interval length in addition to maternal age. As already indicated, these are closely related to maternal age, and they have attracted much attention in the literature on how reproductive factors affect child health, mortality, and well-being (Barclay and Kolk 2017; Black, Devereux, and Salvanes 2005; Kravdal 2018). Birth year is also included in the mortality equation in most of the simulations.

The link between the constant unobserved mortality determinants and the reproductive factors could be established differently, but from the present perspective

that is unimportant. If sibling models with birth year included were to give very 'wrong' estimates using these particular simulated populations, that would be sufficient ground for general concern about this kind of estimation. However, simulation based on other assumptions is carried out in the last part of the analysis, as explained below.

In the second step, sibling models are estimated from the simulated populations, and the resulting effects are compared with those used in the simulation (Section 3). The third step shows that the effects of the different reproductive variables and birth year (and other variables) can be correctly estimated from the simulated populations using a multilevel-multiprocess model, specified just as the one used in the simulation (Section 4). While this might be considered a trivial circularity it confirms that the estimation and simulation procedures are working as they are supposed to, and hopefully strengthens confidence in all parts of the analysis. Also, this check is supplemented by estimations that illustrate how sensitive the results are to the categorization of maternal age. The categorization of maternal age is a potential concern because of the close relationship between this variable and the other reproductive variables. For comparison, it is also shown how important it may be to use a finely specified maternal-age variable when estimating sibling models.

It would be valuable to know whether estimation of a multilevel-multiprocess model such as that described above goes well not only with a population simulated from exactly the same model, but also in other situations. For example, the unobserved fertility determinants may operate in a much more complex way in the real world. One possibility may be that their contribution differs across parity transitions; another could be that there are interactive effects, in the sense that the contribution varies with educational level or other observed characteristics. Also, the unobserved contributions to fertility and mortality may fit poorly with the assumptions about a normal distribution, or actually they may not be uncorrelated with the observed variables. The fourth step of the analysis is to illustrate the implications of the latter two structures, which are deviations from standard assumptions (Section 5). In order to do that the simulation model is changed so that the random effects are no longer normally distributed or no longer uncorrelated with the observed variables, and it is checked whether estimation of a model such as that used in the third step – and which is built on standard assumptions (in available software) about normal random effects uncorrelated with the observed factors – performs well. Finally, there is a discussion of how one should deal with the possibility that reality may be even more complex than reflected by these two deviations from standard assumptions (Section 6), and a few comments are given about the relevance of the analysis from a broader perspective (Section 7).

2. Data and methods

2.1 Data source and cohorts

The effects used in the simulation were derived from the effects in quite simple fertility and mortality equations (specified below), estimated from Norwegian register data for the years up to 2008 for women born in Norway in 1950–1964 and still living in the country when they were 44 years old. For data protection purposes there was only information about their children's year of birth, not month of birth. Women who bore two or more children in the same year, most of whom were probably twins, were excluded from the estimation.

2.2 Finding effects to be used in the simulation

The first step was to estimate the following discrete-time hazard models for first and higher-order births, up to the fifth, between age 17 and maximum age 44:

$$\log \left(p^{(1)}/(1-p^{(1)}) \right) = \beta^{(1)}_0 + \boldsymbol{\beta}^{(1)}_1 \mathbf{A} + \boldsymbol{\beta}^{(1)}_2 \mathbf{Y} + \boldsymbol{\beta}^{(1)}_3 \mathbf{E} + \boldsymbol{\beta}^{(1)}_4 \mathbf{AE} + \boldsymbol{\beta}^{(1)}_5 \, (\text{year}{-}1980)\mathbf{A},$$

and

$$\log \left(p^{(2)}/(1-p^{(2)}) \right) = \beta^{(2)}_0 + \boldsymbol{\beta}^{(2)}_1 \mathbf{A} + \boldsymbol{\beta}^{(2)}_2 \mathbf{Y} + \boldsymbol{\beta}^{(2)}_3 \mathbf{E} + \boldsymbol{\beta}^{(2)}_4 \mathbf{D} + \boldsymbol{\beta}^{(2)}_5 \mathbf{F}.$$

$p^{(1)}$ is the probability of having a first child during the calendar year, while $p^{(2)}$ is the probability of having a nth child during the calendar year given that the mother's parity at the beginning of the year was $n{-}1$, and n is two or higher. The superscripts [(1)] and [(2)] are used similarly for other variables. $\mathbf{A}$ is a vector of dummies corresponding to three-year age categories (17–19, 20–22, etc., up to 38–40) and the four-year category 41–44, with 29–31 as the reference category. Age refers to the age at the end of the year, i.e., the calendar year less the mother's year of birth. $\mathbf{Y}$ is a vector of dummies corresponding to 1967–1969, five-year categories from 1970–1974 to 2000–2004, and 2005–2008 (with 1980–1984 as the reference). $\mathbf{E}$ is a vector of education dummies corresponding to lower secondary, higher secondary, and some or full tertiary education (primary being the reference) according to information from the year when the woman was 44 years old. $\mathbf{AE}$ is the interaction between $\mathbf{A}$ and $\mathbf{E}$, and was included because women who have been in higher education by age 44 typically have had low first-birth probabilities at low ages and relatively high first-birth probabilities at higher ages. Note that the effects of $\mathbf{E}$ also reflect reverse causality, i.e., that births have consequences for

later education (Cohen, Kravdal, and Keilman 2011). If the goal is to learn something about causal effects of education on fertility, a measure of current education rather than education at age 44 should be included. However, full education histories were not available for these cohorts, and because the intention is only to discuss the value of certain methods, the inclusion of education at age 44 should be of no concern.

In addition to a weak general decline in first-birth rates, and thus slight increase in the proportion remaining childless, there has been a shift towards later entry into parenthood: First-birth rates have fallen most sharply at the lowest ages and have increased among the oldest women. For simplicity, this change in the age pattern was specified as a linear interaction between period (minus 1980) and each of the age dummies. $\mathbf{D}$ is a vector of dummies for duration since last previous birth, corresponding to 1, 2, 3 (reference), 4, 5–6, 7–9, and 10 or more years. $\mathbf{F}$ is a vector of dummies for the number of children already born, which is 1 (reference), 2, 3, or 4. The observations were censored at the time of fifth birth or at age 44, whichever came first (recall that only those alive up to age 44 were included and that the last year of observation, 2008, was at age 44 or later). The βs are the corresponding effects.

Also, the following model for the chance (m) of dying within the calendar year after the year of birth was estimated (for each of the children included in the fertility analysis except those born in 2008, as they were not observed through the subsequent year):

$$\log (m/(1-m)) = \gamma_0 + \gamma_1 \mathbf{A} + \gamma_2 \mathbf{B} + \gamma_3 \mathbf{E} + \gamma_4 \mathbf{D} + \gamma_5 \mathbf{O}.$$

The vectors $\mathbf{A}$, $\mathbf{D}$, and $\mathbf{E}$ are as just defined (i.e., $\mathbf{A}$ represents maternal age at the end of the year when the child was born and $\mathbf{D}$ represents the length of the previous birth interval). The $\mathbf{D}$-dummies are set to 0 for first-born children. $\mathbf{O}$ represents the child's birth order, which is mother's parity at the beginning of the year when the child is born (represented by $\mathbf{F}$ in the fertility equation) plus 1. The γs are the corresponding effects. B is the birth year, and for simplicity a linear birth-year effect is assumed.

It is important to note, however, that although these effects of reproductive factors on infant mortality (and which are shown in the first column of all tables) are estimated from real data, they are of little substantive interest. This is because the model is very simple and controls for education only. Nor are the results from the more complex models (sibling models or multilevel-multiprocess models) estimated from the simulated populations substantively interesting. As further explained below, these simulations are based on the estimates from the fertility and mortality models just described and arbitrary assumptions about the variance in and correlation between unobserved contributions to fertility and mortality. To obtain more interesting results, more complex models would have to be estimated from the real data.

2.3 Simulating births and deaths

Half of the women included in the fertility and mortality estimation (224,927 women) were randomly selected and used as the basis population in all simulations. In total, up to 100 simulations of a certain type were carried out for this basis population, each of them creating what is referred to below as a 'simulated population' (see sub-section 2.5 on the number of replications). This population size was chosen because it was close to the maximum that could be handled in the estimation of the multilevel-multiprocess models with the chosen software (see details below). For each of the women, the simulation of births and infant deaths was based on the values of the cohort and education variables, the estimates of the βs in the fertility equation, and a slightly modified version of the estimates of the γs in the mortality equation. Let us call these two sets of effects β^ and γ^.

The estimated maternal-age effects in the mortality equation were modified in two steps. First, effects for all maternal ages 17–44 in one-year categories were defined by setting the effects for age 18, 21, 24, etc., equal to the effects estimated for the corresponding three-year age groups (17–19, 20–22, 23–25, etc.), and then interpolating and extrapolating. This was done because, in reality, infant mortality is not a step-function of maternal age, and it is particularly important to make realistic assumptions about the effects of that variable. Second, some irregularities (partly reflecting the small number of observations in some categories) were smoothed out, because the effects used in the simulation of mortality are shown in tables and compared with results obtained when various models are estimated from the simulated population, and such comparison is simpler when the effects are smoother. Additionally, the effects of interval and birth order were smoothed, and the effect of birth year was set to be somewhat stronger than the one that was estimated.

Furthermore, one random effect (called δ) was added to the two fertility equations and another random effect (called ε) to the mortality equation when simulating. The former represents constant unobserved factors affecting the woman's fertility throughout her reproductive career, and the latter represents constant unobserved factors affecting the mortality of all her children. Both random effects were assumed to be normally distributed with mean 0 and standard deviation 1 (any other number could have been chosen), and the correlation between them was set arbitrarily to 0.5, which can be considered a quite strong correlation. In other words, for each woman in the simulated population, two numbers were drawn (at age 17 when the simulation starts) from a bivariate normal distribution with the mentioned means, standard deviations, and correlation. Thus, the equations used in the simulation were as follows:

$$\log\left(p^{(1)}/(1-p^{(1)})\right) = \hat{\beta}^{(1)}_0 + \hat{\beta}^{(1)}_1 \mathbf{A} + \hat{\beta}^{(1)}_2 \mathbf{Y} + \hat{\beta}^{(1)}_3 \mathbf{E} + \hat{\beta}^{(1)}_4 \mathbf{AE}$$
$$+ \hat{\beta}^{(1)}_5 (\text{year-1980})\mathbf{A} + \delta$$

$$\log (p^{(2)}/(1-p^{(2)})) = \hat{\beta}^{(2)}_0 + \hat{\boldsymbol{\beta}}^{(2)}_1 \mathbf{A} + \hat{\boldsymbol{\beta}}^{(2)}_2 \mathbf{Y} + \hat{\boldsymbol{\beta}}^{(2)}_3 \mathbf{E} + \hat{\boldsymbol{\beta}}^{(2)}_4 \mathbf{D} + \hat{\boldsymbol{\beta}}^{(2)}_5 \mathbf{F} + \delta$$

$$\log (m/(1-m)) \quad = \hat{\gamma}_0 + \hat{\gamma}_1{}^* \mathbf{A}^* + \hat{\gamma}_2 \mathbf{B} + \hat{\gamma}_3 \mathbf{E} + \hat{\gamma}_4 \mathbf{D} + \hat{\gamma}_5 \mathbf{O} + \varepsilon. \qquad (2)$$

The maternal age variable and the corresponding effects in the mortality equation are now marked with * to symbolize the change to one-year categories.

One-year steps were taken in the simulation: For each year of the woman's life between age 17 and 44 her probabilities of having a child that year and losing the child within the subsequent year were predicted. A childbirth, and possibly a death of that child, was 'assigned to' the woman in that year depending on whether two numbers drawn from uniform distributions over [0,1] were lower than the respective predicted probabilities. (The fact that, for simplicity, a death was assigned in the same year as the birth has no implications for the later estimation, where the outcome is considered as death within the year of birth or the subsequent year.)

Birth year was left out of the mortality equation in some introductory simulations, to set up a contrast to the later and main simulations. In another introductory simulation the two random effects were omitted. In the latter case, the simulated number of births in the different cohorts, the average age at first birth, and the variation in these figures across educational categories accorded very well with findings reported elsewhere (e.g., Kravdal and Rindfuss 2008). This indicates that the estimation of fertility models in the first step and the fertility part of the simulation was done correctly.

2.4 Estimation of models from the simulated populations

The multilevel-multiprocess models that were estimated were specified exactly as model (2) in the simulation, except that it was experimented with some alternative categorizations of **A.** A few different categorizations of **A** were also used in the sibling models, and in some of them birth year was included as a categorical variable, or even left out. All this is specified in detail below.

The estimation of sibling models was carried out with SAS software, as was the simulation, the estimation of effects to be used in the simulation, and the construction of data (from the simulated populations) to be used as input to estimate the multilevel-multiprocess models. The latter estimation was done with the aML software (Lillard and Panis 2003). Other software also exists for such purposes. In particular, there are modules in Stata (Bartus and Rodman 2014) or MLwiN (see an example by Väisänen 2017) that may be used.

In aML, it is an in-built assumption that the random effects are uncorrelated with the observed variables and that they are normally distributed. The normal distributions are approximated by a certain number of 'support points'. Experimentation showed that

it was sufficient to use 20 support points. Using 30 support points gave exactly the same estimates, and (as described below) the estimates (averaged over several replications) from a multilevel model that corresponded exactly to the one used in the simulation were almost identical to the true effects when 20 support points were used. The typical CPU-time for the estimation of one multilevel-multiprocess model from a simulated population was about 1 hour. As mentioned, the simulated population could not be much larger (given the number of variables considered). If it were about 20% larger, estimation would be impossible because of insufficient computer memory. (Recall, however, that the simulated population included as many as 224,927 women, and therefore would be considered large by most standards.)

2.5 Number of replications

Most of the analysis was based on 100 simulated populations of the size just mentioned. Models were estimated from all these, after which the average over the 100 sets of estimates was calculated. This procedure was quite cumbersome when multilevel-multiprocess models were estimated, because of the switching between two types of software (SAS and aML), the long CPU-time, and the large data sets used in the estimation, and it was necessary to break the process into five rounds with 20 simulated populations. Fortunately, the results were very similar when 10 simulated populations were used instead – reflecting the large size of these populations. For simplicity, the last part of the analysis involving multilevel-multiprocess models was therefore based on only 10 simulated populations.[2]

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[2] Note that this simulation-estimation experiment might have been based on simpler fertility equations in the simulation and subsequent estimation of multilevel-multiprocess models. For example, the interactions between age and education and between age and period in the equation for first births might have been left out, or simpler effects of age or duration since previous birth on the birth rates might have been assumed, such as step functions with very few steps. The pattern in the simulated births – when the random effects were excluded from the simulation – would no longer be as close to that observed in the real world, but that might be acceptable from a perspective such as that of the present study. In fact, when the mentioned interactions were dropped in supplementary analysis, the estimates (not shown in tables) from the simulated populations were very similar to those reported below. With simpler models, aML could handle larger simulated populations, and less computer time would be needed for a certain number of replications with the same simulated population size as in the present analysis. However, in an empirical study where the intention is to estimate effects in real data using a multilevel-multiprocess model, it would make sense to include fertility equations that reflect the patterns expected in the 'real' world (which may be quite complex) as much as possible, as this might be important for the results.

2.6 Simulations with other assumptions

In a final step, alternative simulations were carried out where it was no longer assumed that random effects were normally distributed and uncorrelated with the observed factors, in order to see how wrong the estimates from a multilevel-multiprocess model (still with standard assumptions about the random effects) would then be. In some of these alternative simulations, a variable G was added:

$$\log (p^{(1)}/(1-p^{(1)})) = \beta\widehat{\ }^{(1)}_0 + \boldsymbol{\beta}\widehat{\ }^{(1)}_1 \mathbf{A} + \boldsymbol{\beta}\widehat{\ }^{(1)}_2 \mathbf{Y} + \boldsymbol{\beta}\widehat{\ }^{(1)}_3 \mathbf{E} + \boldsymbol{\beta}\widehat{\ }^{(1)}_4 \mathbf{AE}$$
$$+ \boldsymbol{\beta}\widehat{\ }^{(1)}_5 (\text{year-1980})\mathbf{A} + \beta^{(1)}_6 G + \delta$$
$$\log (p^{(2)}/(1-p^{(2)})) = \beta\widehat{\ }^{(2)}_0 + \boldsymbol{\beta}\widehat{\ }^{(2)}_1 \mathbf{A} + \boldsymbol{\beta}\widehat{\ }^{(2)}_2 \mathbf{Y} + \boldsymbol{\beta}\widehat{\ }^{(2)}_3 \mathbf{E} + \boldsymbol{\beta}\widehat{\ }^{(2)}_4 \mathbf{D} + \boldsymbol{\beta}\widehat{\ }^{(2)}_5 \mathbf{F}$$
$$+ \beta^{(2)}_6 G + \delta$$
$$\log (m/(1-m)) = \gamma\widehat{\ }_0 + \boldsymbol{\gamma}\widehat{\ }_1{}^* \mathbf{A}^* + \boldsymbol{\gamma}\widehat{\ }_2 \mathbf{B} + \boldsymbol{\gamma}\widehat{\ }_3 \mathbf{E} + \boldsymbol{\gamma}\widehat{\ }_4 \mathbf{D} + \boldsymbol{\gamma}\widehat{\ }_5 \mathbf{O} + \gamma_6 G + \varepsilon. \quad (3)$$

G was a number drawn randomly for each woman from a standard normal distribution and made to correlate with ε, and thus δ. Other simulations instead included H or K, which were defined as 1 if G was larger than 0 or 0.67, respectively, and 0 otherwise. In one set of estimations of multilevel-multiprocess models, G, H, or K were included in the models to examine the implications of having an observed determinant of fertility and mortality (with one of three different distributions) that was correlated with the unobserved normally distributed contributions to mortality and fertility (i.e., ε and δ). In another set of estimations, H and K were not included in the models, but were instead considered as adding to the unobserved contributions to mortality and fertility, which were then no longer normally distributed. H and K did not have to be correlated with ε and δ in this illustration of the implications of deviations from normality; it was just convenient to use the same variables in this part of the analysis. In a supplementary step a version of H that was uncorrelated with ε and δ was used instead, and this gave very similar results. See further description of these simulation-estimation experiments below.

3. Estimates from sibling models

Estimates from various sibling models are presented first, while those from multilevel-multiprocess models are presented in the next sections. Table 1 gives an overview of the simulations and estimations and the conclusions that can be drawn from them.

Table 1:   Overview of simulations, estimations, and results

	Assumptions in simulation if different from equation (2)	Features of the estimation	Conclusions: Estimated effects of maternal age as effects in simulation?
Table 2	0 effect of birth year	Sibling model: 1-year categories for maternal age	Yes
		Sibling model: Broader categories	No
Table 3		Sibling model: Linear effect of birth year and various categorizations of maternal age	No
Figure 1, Appendix Table A-1		Sibling model: Various categorizations of birth year and maternal age	No
Table 4		Multilevel-multiprocess model: 1-year categories for maternal age	Yes
Table 5		Multilevel-multiprocess model: Broader categories for maternal age	No, but smaller differences than in Table 2
Table 6	G (normal) or H (dichotomous) correlated with unobserved contributions to fertility and mortality are included	Multilevel-multiprocess model: G or H included (i.e., considered as observed)	Yes
	Goal: see what happens if unobserved variables are correlated with observed		
Table 7	H or K (both dichotomous) correlated with unobserved contributions to fertility and mortality are included	Multilevel-multiprocess model: H or K not included (i.e., considered as unobserved)	Yes
	Goal: see what happens if unobserved variables are not normally distributed		

3.1 Estimation based on a simulated population where mortality was assumed to be unaffected by birth year

In the first step, the simulations were carried out without including birth year in the mortality equation. As expected, the effects estimated from an ordinary logistic model, i.e., a model without sibling fixed effects, were very different from the effects used in the simulation (not shown in tables). In particular, the mortality disadvantage among children of high birth order and children born to young mothers was overestimated. This reflects the fact that the mothers of these children tend to have experienced high birth rates as a 'result of' a high value of the fertility random effect, which in turn is linked to a high value of the mortality random effect. In other words, being in these categories reflects unobserved characteristics of the mothers that also produce high infant mortality. However, and also as expected, the estimates from the sibling model (Table 2, column 2) were very close to the true effects (Table 2, column 1). In other words, in the hypothetical situation where there is no effect of birth year in real life (and such an effect is not included in the model that is estimated either) there seems to be no problem. However, maternal age must be finely specified to obtain a correct separation of the effects of the three highly correlated reproductive variables.

Table 2: **Averages of effects on infant mortality estimated from 100 simulated populations, when the effect of birth year is set to 0 in the simulations**

	Effects used in the simulation[a]	Estimated effects in sibling fixed-effects logistic model[b]			
		Size of categories for maternal age			
		1 year	2 years	3 years	5 years
Maternal age					
17	0.66	0.66	0.54	0.42	0.23
18	0.60	0.60			
19	0.54	0.54	0.43		
20	0.48	0.48		0.30	
21	0.42	0.42	0.33		
22	0.36	0.36			0.09
23	0.30	0.30	0.23	0.16	
24	0.24	0.24			
25	0.18	0.18	0.12		
26	0.13	0.13		0.05	
27	0.09	0.09	0.05		0
28	0.05	0.05			
29	0.02	0.02	0	0	
30	0	0			
31	−0.02	−0.02	−0.02		
32	−0.04	−0.04		−0.03	0.00
33	−0.06	−0.06	−0.06		
34	−0.08	−0.08			
35	−0.08	−0.08	−0.07	−0.03	
36	−0.08	−0.08			
37	−0.08	−0.09	−0.07		0.05
38	−0.08	−0.08		0.00	
39	−0.08	−0.09	−0.05		
40	−0.08	−0.07			
41	−0.08	−0.09	−0.06	0.00	
42	−0.08	−0.10			0.08
43	−0.08	−0.09	−0.06		
44	−0.08	−0.09		0.01	
Previous interval					
1	0.57	0.58	0.58	0.59	0.61
2	0.07	0.07	0.08	0.08	0.09
3	0	0	0	0	0
4	−0.05	−0.05	−0.05	−0.06	−0.07
5–6	−0.12	−0.12	−0.13	−0.14	−0.16
7–9	−0.12	−0.12	−0.14	−0.17	−0.20
10+	−0.12	−0.12	−0.16	−0.21	−0.29
Birth order					
1	0	0	0	0	0
2	0.10	0.10	0.08	0.05	0.00
3	0.20	0.20	0.16	0.11	0.02
4	0.30	0.30	0.24	0.17	0.05
5	0.40	0.40	0.33	0.23	0.09

Notes: [a] The model used in the simulation was as explained in the text. The effect of birth year on infant mortality was assumed to be 0. [b] The model included the variables shown in this table plus sibling fixed effects. The estimates that are shown for age x are for a category including ages x to x+1 (third column), x+2 (fourth column), or x+4 (fifth column), except that the highest age category only includes ages up to 44.

When two-year categories for maternal age were used (Table 2, column 3) the effects of maternal age and birth order were about 20% weaker (judging by the difference in mortality between the lowest and highest age group) than when one-year categories were used. Further changes in the same direction occurred when the maternal-age categories were increased to three or five years (Table 2, columns 4 and 5). In fact, when five-year categories were used, the estimated effects of maternal age and birth order were about 80% weaker than the corresponding true values. The effects of the birth interval length became stronger as the effects of birth order and maternal age became weaker.

3.2 Estimation based on simulated populations where a linear effect of birth year on mortality was assumed

When a linear effect of birth year was included in the mortality equation in the simulation, the estimates from sibling models (now including birth year) were no longer close to the true values, even when small categories for maternal age were used.

Consider first the results obtained when a linear effect of birth year was assumed in the estimation. Then, it was impossible to estimate a model with one-year categories for maternal age; instead, two-year, three-year, or five-year categories were used. Actually, the estimates did not vary much across these three specifications (Table 3). In all cases the negative linear effect of birth year was strongly overestimated, and a strongly positive effect of maternal age appeared. The actual decline in infant mortality with increasing maternal age was, so to speak, attributed to birth year. However, the effects of birth order and birth interval were never very different from the true values – unlike the pattern that appeared (see Table 2) when birth year was kept out of the simulation and estimation and gradually broader categories of maternal age were used in the estimation.

Table 3: **Averages of effects on infant mortality estimated from 100 simulated populations, using different categorizations of maternal age in the estimation**

	Effects used in the simulation[a]	Estimated effects in sibling fixed-effects logistic model[b]		
		Size of categories for maternal age		
		2 years	3 years	5 years
Maternal age				
17	0.66	0.10	0.06	0.06
18	0.60			
19	0.54	0.07		
20	0.48		0.02	
21	0.42	0.03		
22	0.36			0.00
23	0.30	0.00	−0.02	
24	0.24			
25	0.18	−0.03		
26	0.13		−0.04	
27	0.09	−0.03		0
28	0.05			
29	0.02	0	0	
30	0			
31	−0.02	0.05		
32	−0.04		0.07	0.11
33	−0.06	0.09		
34	−0.08			
35	−0.08	0.16	0.18	
36	−0.08			
37	−0.08	0.24		0.29
38	−0.08		0.32	
39	−0.08	0.35		
40	−0.08			
41	−0.08	0.41	0.43	
42	−0.08			0.47
43	−0.08	0.50		
44	−0.08		0.51	
Birth year	−0.02	−0.063	−0.063	−0.061
Previous interval				
1	0.57	0.58	0.58	0.58
2	0.07	0.07	0.07	0.08
3	0	0	0	0
4	−0.05	−0.05	−0.05	−0.05
5–6	−0.12	−0.12	−0.12	−0.12
7–9	−0.12	−0.12	−0.12	−0.12
10+	−0.12	−0.12	−0.12	−0.13
Birth order				
1	0	0	0	0
2	0.10	0.10	0.09	0.08
3	0.20	0.19	0.19	0.16
4	0.30	0.30	0.29	0.26
5	0.40	0.40	0.39	0.35

Notes: [a] The model used in the simulation was as explained in the text. [b] The model included the variables shown in this table plus sibling fixed effects. The estimates that are shown for age x are for a category including ages x to x+1 (second column), x+2 (third column), or x+4 (fourth column), except that the highest age category only includes ages up to 44.

In the second set-up, birth year was included in the regression models as a categorical variable, and a few different categorizations were tried. One-year categories cannot be specified for both maternal age and birth year, but various other combinations of one-, two-, three-, and five-year categories were experimented with. When one-year categories were specified for maternal age and two-year categories were specified for birth year, very sharp effects of maternal age appeared, while there was essentially no effect of birth year (see plots of maternal age effects in Figure 1; effects of all variables are shown in Table A-1). In the opposite situation, with two-year categories for maternal age and one-year categories for birth year, the effect of maternal age was positive, while there was a strongly negative effect of birth year. The estimates were closer to the true values when using two-year categories for both maternal age and birth year, although – most importantly – the estimated negative effect of maternal age was too weak (only about 60% of the true effect). The effect of birth year was too strongly negative compared to the true value of –0.02 per year, which corresponds to –0.82 over the 41-year period. A similar pattern appeared when three-year categories were used for both maternal age and birth year (not shown in figure or table).
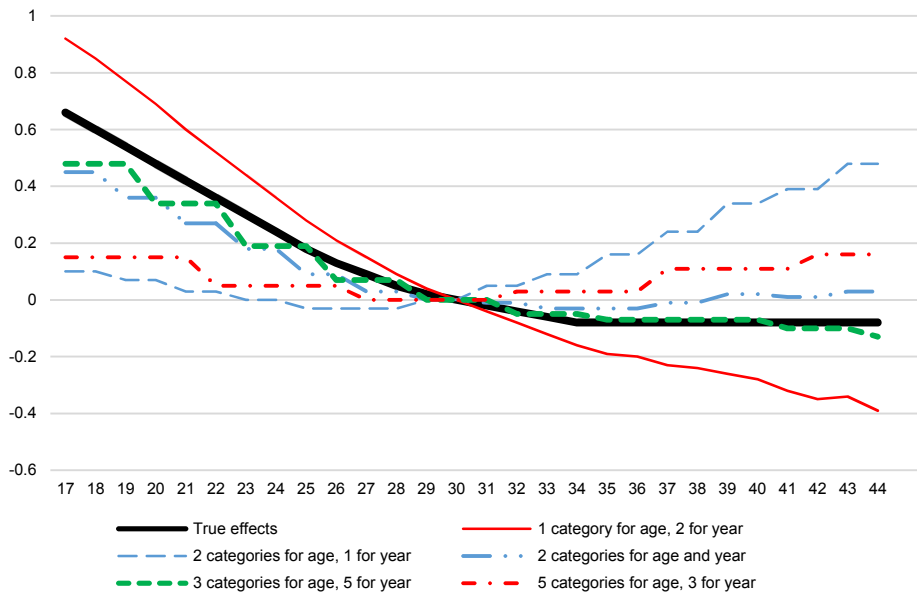
Figure 1: **Averages of effects of maternal age on infant mortality estimated from 100 simulated populations, using different categorizations of maternal age and birth year in the estimation**

When three-year categories were used for maternal age and five-year categories were used for birth year the effect of maternal age was again too weakly negative, but the deviation from the true effect was smaller than with the other specifications (the difference in mortality between the lowest and highest age being about 20% smaller than the corresponding true values). With this specification the effect of birth year was also too weak, and the effect of birth order was strongly underestimated. Using instead five-year categories for maternal age and three-year categories for birth year, a U-shaped effect of maternal age appeared, while the effect of birth order became smaller than in any of the other models, and the effect of birth year again became too negative.

Note that the differences in results across various categorizations of maternal age do not just reflect the problem related to separation of effects of the reproductive variables that appeared when birth year was kept out of the simulation and estimation (shown in Table 2). Patterns similar to those shown in Figure 1 and Table A-1 were also found when birth order and birth interval were excluded from the simulation and estimation (not shown in tables). In this alternative set-up, the estimates obtained with three-year categories for maternal age and five-year categories for birth year were still those that came closest to the true values, and they were now actually even closer. With the other categorizations, effects of maternal age remained almost unchanged when birth order and birth interval effects were excluded, while birth year effects became more strongly negative.

If birth year was included when doing the simulation, but not in the estimated model, the effect of maternal age captured – as one would expect – the sum of the true effect of maternal age and the effect of birth year. For example, using one-year categories for maternal age, the difference in mortality between the highest and lowest age was –1.30 (Table A-2). This is not very different from the corresponding true effect of maternal age (–0.74) plus 27 times (because age 44–17 is 27) the effect of a one-year increase in birth year (–0.02). If three-year categories of maternal age were used instead, the effects of both maternal age and birth order were weaker, while the effect of birth interval length was stronger, just as seen in the situation where there was no birth-year effect (Table 2). Thus, under the assumption that there is a birth-year effect in reality, the lesson is that birth year should be left out of the estimated model and small categories used for maternal age. The estimated effects of maternal age will then reflect well the combined effect of maternal age and year, and the effects of birth order and birth interval will be correctly estimated.

4. Estimates from multilevel-multiprocess models

Estimates from multilevel-multiprocess models are shown in Table 4. They are very close to the true values, regardless of whether 10 or 100 replications are used, which, as mentioned, justifies the use of 10 replications in the remaining analysis.

Table 4: Averages of effects on infant mortality estimated from 10 or 100 simulated populations, using a multilevel-multiprocess model in the estimation

	Effects used in simulation[a]	Estimates from multilevel–multiprocess model[b]	
		Average over 10 simulated populations	Average over 100 simulated populations
Maternal age			
17	0.66	0.67	0.66
18	0.60	0.59	0.61
19	0.54	0.55	0.54
20	0.48	0.48	0.48
21	0.42	0.42	0.42
22	0.36	0.38	0.36
23	0.30	0.30	0.30
24	0.24	0.25	0.24
25	0.18	0.19	0.18
26	0.13	0.13	0.13
27	0.09	0.11	0.09
28	0.05	0.05	0.05
29	0.02	0.02	0.02
30	0	0	0
31	−0.02	−0.01	−0.02
32	−0.04	−0.02	−0.04
33	−0.06	−0.06	−0.06
34	−0.08	−0.06	−0.08
35	−0.08	−0.08	−0.09
36	−0.08	−0.07	−0.08
37	−0.08	−0.04	−0.08
38	−0.08	−0.05	−0.08
39	−0.08	−0.05	−0.07
40	−0.08	−0.07	−0.07
41	−0.08	−0.08	−0.09
42	−0.08	−0.07	−0.11
43	−0.08	−0.09	−0.09
44	−0.08	−0.07	−0.10
Birth year	−0.02	−0.020	−0.020
Previous interval			
1	0.57	0.57	0.57
2	0.07	0.06	0.07
3	0	0	0
4	−0.05	−0.06	−0.05
5–6	−0.12	−0.12	−0.12
7–9	−0.12	−0.13	−0.12
10+	−0.12	−0.13	−0.12

Table 4: **(Continued)**

	Effects used in simulation[a]	Estimates from multilevel–multiprocess model[b]	
		Average over 10 simulated populations	Average over 100 simulated populations
Birth order			
1	0	0	0
2	0.10	0.11	0.10
3	0.20	0.20	0.20
4	0.30	0.30	0.30
5	0.40	0.39	0.40
Education			
10	0	0	0
11	−0.08	−0.09	−0.08
12–13	−0.16	−0.17	−0.16
14+	−0.24	−0.25	−0.24
Std. fertility random term	1.0	1.00	1.00
Std. mortality random term	1.0	1.00	1.00
Corr. random terms	0.5	0.51	0.50

Notes: [a] The model used in the simulation was as explained in the text. [b] The equation for mortality included the variables shown in this table, and there were also equations for fertility, as explained in the text.

When broader categories for maternal age were used in the estimation, the effects of that variable and of birth order became weaker (while the effect of birth year became stronger). This is shown in Table 5. However, the changes in this direction were much less pronounced than when a sibling model was estimated and mortality was assumed to be unaffected by the birth year in the simulation (Table 2), or when a birth-year effect was included in the simulation but not in the estimation, so that the maternal-age effect picks up a combination of the maternal-age effect and the birth-year effect (Table A-2).

Table 5: Averages of effects on infant mortality estimated from 10 simulated populations, using a multilevel-multiprocess model in the estimation and different categorizations of maternal age

	Effects used in simulation[a]	Estimated effects in multilevel–multiprocess models[b]		
		Size of categories for maternal age		
		2 years	3 years	5 years
Maternal age				
17	0.66	0.59	0.53	0.35
18	0.60			
19	0.54	0.46		
20	0.48		0.37	
21	0.42	0.36		
22	0.36			0.14
23	0.30	0.24	0.21	
24	0.24			
25	0.18	0.13		
26	0.13		0.08	
27	0.09	0.06		0
28	0.05			
29	0.02	0	0	
30	0			
31	−0.02	−0.05		
32	−0.04		−0.04	−0.05
33	−0.06	−0.07		
34	−0.08			
35	−0.08	−0.09	−0.06	
36	−0.08			
37	−0.08	−0.08		−0.02
38	−0.08		−0.03	
39	−0.08	−0.07		
40	−0.08			
41	−0.08	−0.11	−0.08	
42	−0.08			−0.02
43	−0.08	−0.14		
44	−0.08		−0.08	
Birth year	−0.02	−0.021	−0.022	−0.026
Previous interval				
1	0.57	0.57	0.57	0.58
2	0.07	0.07	0.07	0.08
3	0	0	0	0
4	−0.05	−0.05	−0.05	−0.05
5–6	−0.12	−0.13	−0.14	−0.14
7–9	−0.12	−0.12	−0.13	−0.13
10+	−0.12	−0.12	−0.13	−0.15

Table 5:   (Continued)

	Effects used in simulation[a]	Estimated effects in multilevel–multiprocess models[b]		
		Size of categories for maternal age		
		2 years	3 years	5 years
Birth order				
1	0	0	0	0
2	0.10	0.10	0.09	0.06
3	0.20	0.19	0.18	0.13
4	0.30	0.29	0.26	0.20
5	0.40	0.39	0.36	0.28
Education				
10	0	0	0	0
11	−0.08	−0.08	−0.08	−0.10
12–13	−0.16	−0.16	−0.17	−0.18
14+	−0.24	−0.24	−0.24	−0.27
Std. fertility random term	1.0	1.00	1.00	1.00
Std. mortality random term	1.0	1.00	1.00	1.01
Corr. random terms	0.5	0.51	0.51	0.53

Notes: [a] The model used in the simulation was as explained in the text. [b] The equation for mortality included the variables shown in this table, and there were also equations for fertility, as explained in the text. The estimates that are shown for age x are for a category including ages x to x+1 (second column), x+2 (third column), or x+4 (fourth column), except that the highest age category only includes ages up to 44.

If birth year was not included in the mortality equation in the multilevel-multiprocess model, the estimated effect of maternal age (difference between mortality in lowest and highest age) was almost the same as the true effect of maternal age plus 27 times the effect of birth year (not shown in tables). In other words – and as expected – the birth year effect was captured by the maternal-age effect, just as with the sibling analysis.[3]

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[3] Note that because some combinations of maternal age and birth order are impossible or very rare, the use of a variable that combines these two may facilitate the discussion of the results. Using such a variable also means that effects of maternal age are allowed to vary across birth orders. (Alternatively, one might predict contributions to mortality for possible combinations of maternal age and birth order from main effects models such as estimated here, or from models also including an interaction between maternal age and birth order.) The conclusions are the same in this case: Such combined effects of maternal age and birth order in a multilevel-multiprocess model can be estimated even in the presence of an effect of birth year (as done by Kravdal 2018), but if they are estimated with a sibling approach one should not try to separate out the effect of birth year. Instead, it must be accepted that the variation in mortality across the maternal-age dimension reflects effects of both maternal age and birth year.

5. A reality that deviates from the standard assumptions of the multilevel-multiprocess model

So far, it has been assumed in the simulations that the random effects, which reflect unobserved constant mother-level contributions to mortality and fertility, are normally distributed and uncorrelated with the observed variables (except that the unobserved contribution to mortality is correlated with the mortality determinants that are a result of fertility because of the correlation between the unobserved contributions to fertility and mortality). These assumptions were also built into the estimation of multilevel-multiprocess models presented in the foregoing section. How would the estimation of such models go if these two assumptions fit poorly with reality? Examples of this are given below by carrying out alternative simulations with other assumptions about the random effects, and then estimating the models similar to those above.

5.1 An additional variable correlated with the unobserved contributions to fertility and mortality

The first step was to create a population where the normally distributed unobserved contributions to fertility and mortality are correlated with a variable considered as an observed determinant. This was done by adding a standard normally distributed variable G that was strongly related to the mortality random effect (correlation 0.70) – and therefore also to the fertility random effect – to both the mortality and fertility equations in the simulation, as shown in (3). G was considered as an observed variable and assumed to have large effects: The coefficients $\beta^{(1)}_6$ and $\beta^{(2)}_6$ in the two fertility equations were arbitrarily set to 1.1 (i.e., the log-odds of having a child is 1.1 higher when G = 1 than when G = 0), and the coefficient γ_6 in the mortality equation was set to 0.8.

It turned out that when G was included (as an observed variable) in the fertility and mortality equations in the estimation, the estimates of the effects of the reproductive variables (Table 6, column 2) were very close to the true values. However, the estimated standard deviations of the mortality and fertility random effects, and the correlation between them, were smaller than the values used in the simulations. Furthermore, the estimated effects of G were different. For example, the effect of G on mortality was 1.51. This is not far from the sum of the correct 0.8 plus the coefficient obtained by regressing the unobserved contribution to mortality (ε) on G (0.66; not shown in the tables). Intuitively, 0.8 G plus the unobserved contribution to mortality (ε) may be thought of as being the same as $(0.8 + 0.66)$ G plus something that is

uncorrelated with G. The latter may be considered as represented by the mortality random effect in the model that is estimated.

Table 6: **Averages of effects on infant mortality estimated from 10 simulated populations, using a multilevel-multiprocess model, and when a variable (G or H) that is correlated with the unobserved contributions to fertility and mortality is included in the simulation and estimation**

	Effects used in simulation[a]	Estimates from multilevel-multiprocess model[b]	
		G included in simulation and estimation	H included in simulation and estimation
Maternal age			
17	0.66	0.67	0.67
18	0.60	0.59	0.61
19	0.54	0.53	0.54
20	0.48	0.48	0.49
21	0.42	0.42	0.42
22	0.36	0.36	0.37
23	0.30	0.30	0.32
24	0.24	0.24	0.25
25	0.18	0.17	0.19
26	0.13	0.14	0.14
27	0.09	0.08	0.10
28	0.05	0.05	0.06
29	0.02	0.01	0.02
30	0	0	0
31	−0.02	−0.01	−0.01
32	−0.04	−0.03	−0.03
33	−0.06	−0.04	−0.06
34	−0.08	−0.08	−0.07
35	−0.08	−0.08	−0.09
36	−0.08	−0.08	−0.08
37	−0.08	−0.08	−0.08
38	−0.08	−0.07	−0.07
39	−0.08	−0.10	−0.10
40	−0.08	−0.09	−0.09
41	−0.08	−0.08	−0.12
42	−0.08	−0.09	−0.09
43	−0.08	−0.08	−0.08
44	−0.08	−0.04	−0.06
Birth year	−0.02	−0.020	−0.020
Previous interval			
1	0.57	0.57	0.58
2	0.07	0.07	0.07
3	0	0	0
4	0.05	−0.05	−0.04
5–6	−0.12	−0.12	−0.11
7–9	−0.12	−0.12	−0.11
10+	−0.12	−0.14	−0.12

Table 6:   **(Continued)**

	Effects used in simulation[a]	Estimates from multilevel-multiprocess model[b]	
		G included in simulation and estimation	H included in simulation and estimation
Birth order			
1	0	0	0
2	0.10	0.10	0.10
3	0.20	0.20	0.20
4	0.30	0.29	0.29
5	0.40	0.39	0.39
Education			
10	0	0	0
11	−0.08	−0.08	−0.08
12–13	−0.16	−0.16	−0.17
14+	−0.24	−0.24	−0.25
G	0.8	1.51	
H	0.8		1.94
Std. fertility random term	1.0	0.94	0.96
Std. mortality random term	1.0	0.71	0.84
Corr. random terms	0.5	0.38	0.42

Notes: [a] The model used in the simulation was as explained in the text, except that either G or H was added to the mortality equation and both fertility equations. G was drawn from a standard normal distribution whose correlation with the mortality random term was 0.7. H was set to 1 if G was larger than 0, and otherwise 0. The effects of G and H in the fertility equations were 1.1, while they were 0.8 in the mortality equation. [b] The equation for mortality included the variables shown in this table, and there were also equations for fertility, as explained in the text. The fertility equations also included G or H.

In an alternative set-up, the covariate correlated with the unobserved variable was not normally distributed, but dichotomous. To be more specific, H was included in a simulation instead of G. It was defined as 1 if G was larger than 0, and 0 otherwise, and it was assumed to have the same effects on fertility and mortality as G. (Introducing H thus means that 0.8 is added to the log-odds of dying and 1.1 to the log-odds of having a child for half of the individuals.) When H was included in the estimation model instead of G, the estimated standard deviations of the mortality and fertility random effects, and the correlation between them, were smaller than the values used in the simulations, while the effects of the reproductive variables were very close to the true values. This is just as found in the simulation-estimation experiment involving G. Using the variable K in the same way instead of H or G, which means that 0.8 and 1.1 were added to mortality and fertility for only about one-quarter of the individuals, gave almost the same results.

5.2 An additional unobserved factor that is not normally distributed

The second main step was to create a situation where the unobserved factor was not normally distributed. This was achieved in a simple way by using the dichotomous

variables H or K instead of G in the simulation, with effects on fertility and mortality such as those just mentioned. These variables were then considered as unobserved, and therefore kept out of the regression equations. When H was used, the estimated effects of maternal age and birth order were somewhat stronger than assumed in the simulation: The coefficients for the 10 highest age groups were on average 0.02 'too low', while those for teenagers were 0.03 'too high' (Table 7, column 2). Also, the birth order effects were sharper (12% for the highest birth order) than assumed in the simulation. The standard deviations of the fertility and mortality random effects were larger than 1 and the correlation between them larger than 0.5. An estimated correlation larger than 0.5 makes good sense, because the mortality equation in the model that is estimated may be considered as including a random effect that captures the sum of the 'original' normally distributed unobserved contribution to mortality (ε in the simulation equation) and another unobserved factor H (which is dichotomous and correlated with it) multiplied by its mortality effect, while the same H multiplied with its fertility effect is added to the 'original' normally distributed unobserved contribution to fertility (δ in the simulation equation). When K was included in the simulation instead of H the effects of maternal age and birth order were marginally weaker than the true values: The effects for the teenagers were 0.03 'too low' (Table 7, column 3).

To conclude, estimates close to the true values were obtained in these particular situations where the unobserved contributions to fertility and mortality (δ and ε) and an observed fertility and mortality determinant (G, H, or K) were correlated in the simulated population, or where the unobserved contributions to fertility and mortality (H or K multiplied by the corresponding effects plus δ or ε) were not normally distributed. However, this does not mean that there are never problems when a multilevel-multiprocess model is estimated based on standard assumptions about normally distributed random effects uncorrelated with the observed variables, and these two assumptions fit poorly with the real world.

Table 7: **Averages of effects on infant mortality estimated from 10 simulated populations, using a multilevel-multiprocess model in the estimation, and when a variable (H or K) that is correlated with the unobserved contributions to fertility and mortality and is not normally distributed is included in the simulation, but not in the estimation**

	Effects used in simulation[a]	Estimates from multilevel-multiprocess model[b]	
		H included in simulation	K included in simulation
Maternal age			
17	0.66	0.70	0.63
18	0.60	0.63	0.57
19	0.54	0.56	0.51
20	0.48	0.52	0.46
21	0.42	0.43	0.40
22	0.36	0.38	0.33
23	0.30	0.32	0.27
24	0.24	0.25	0.22
25	0.18	0.19	0.17
26	0.13	0.13	0.12
27	0.09	0.10	0.09
28	0.05	0.06	0.04
29	0.02	0.02	0.02
30	0	0	0
31	−0.02	−0.01	−0.01
32	−0.04	−0.03	−0.04
33	−0.06	−0.06	−0.06
34	−0.08	−0.07	−0.08
35	−0.08	−0.10	−0.06
36	−0.08	−0.09	−0.07
37	−0.08	−0.09	−0.08
38	−0.08	−0.08	−0.10
39	−0.08	−0.12	−0.06
40	−0.08	−0.10	−0.02
41	−0.08	−0.14	−0.11
42	−0.08	−0.11	−0.14
43	−0.08	−0.10	−0.09
44	−0.08	−0.08	−0.03
Birth year	−0.02	−0.019	−0.020

Table 7:   (Continued)

	Effects used in simulation[a]	Estimates from multilevel-multiprocess model[b]	
		H included in simulation	K included in simulation
Previous interval			
1	0.57	0.55	0.57
2	0.07	0.06	0.07
3	0	0	0
4	−0.05	−0.04	−0.04
5–6	−0.12	−0.11	−0.13
7–9	−0.12	−0.12	−0.14
10+	−0.12	−0.13	−0.14
Birth order			
1	0	0	0
2	0.10	0.13	0.09
3	0.20	0.25	0.19
4	0.30	0.35	0.29
5	0.40	0.45	0.39
Education			
10	0	0	0
11	−0.08	−0.07	−0.08
12–13	−0.16	−0.16	−0.16
14+	−0.24	−0.23	−0.24
Std. fertility random term	1.0	1.26	1.29
Std. mortality random term	1.0	1.16	1.33
Corr. random terms	0.5	0.69	0.75

Notes: [a] The model used in the simulation was as explained in the text, except that either H or K was added to the mortality equation and both fertility equations. H and K were derived from a factor G that was drawn from a standard normal distribution whose correlation with the mortality random term was 0.7. H was set to 1 if G was larger than 0, and otherwise 0. K was set to 1 if G was larger than 0.67, and otherwise 0. The effects of H and K in the fertility and mortality equations were 1.1 and 0.8 respectively. [b] The equation for mortality included the variables shown in this table, and there were also equations for fertility, as explained in the text.

6. Summary and conclusion

6.1 Sibling model

There is considerable interest in learning about the effects of maternal (or paternal) age on various aspects of child health and other child outcomes, and since there are many factors of importance for a woman's fertility that also affect her children's lives, it makes good sense to estimate sibling fixed-effects models. By doing that, one at least controls for unobserved health determinants that are shared between siblings and that are somehow linked to the mother's reproductive behaviour. Birth order and birth interval may be included along with maternal age, although here it is illustrated that finely specified variables must be used in order to properly separate the effects of these strongly correlated reproductive variables.

However, in such a model the estimated effect of maternal age reflects the sum of the effect of birth year and the more or less 'pure' effect of maternal age (depending on whether there are also other control variables that should be included). That may be quite acceptable for some purposes, but there are also good reasons for wanting to separate these effects. Unfortunately, that is not possible – as can be shown mathematically, and as illustrated in the simulation experiments reported here. Attempts to identify separate effects of maternal age and birth year in a sibling analysis can be very misleading.

6.2 The multilevel-multiprocess model as an alternative, but with some disadvantages

Estimation of a multilevel-multiprocess model may be tried as an alternative, if the required data is available. Such a model may – as in this analysis – be based on the assumption that there are certain unobserved factors that have the same impact on all siblings' mortality or other outcomes of interest (as also assumed in the sibling fixed-effects approach) and a constant effect on the woman's fertility, and that these factors can be well reflected by normally distributed fertility and mortality random effects that are correlated with each other. The normality assumption is standard and built into available software, and it is also typically assumed that the random effects are uncorrelated with the observed variables in the model, except that the mortality random effect is correlated with the factors that reflect fertility (in this case maternal age, birth order, and birth interval length).

The simulation experiments showed that estimates might be quite correct even if the assumptions about random effects being normally distributed and uncorrelated with observed variables fit poorly with reality, although one should be careful when generalizing from the examples. Perhaps the estimation would perform less well in situations with larger deviation from normality or stronger correlations with observed variables, or larger effects of the latter.

More importantly, however, the other mentioned assumptions may reflect reality inadequately. One possibility is that there are actually different unobserved fertility determinants for different parity transitions (although they would perhaps be correlated). This situation may be handled if the contribution from the unobserved determinants is at least quite similar within pairs of transitions. Reality may then be represented well enough (with the aML software) by, for example, including one equation for first and second births (or a separate for first, as in the present study) and one for third and higher-order births, specifying one random effect common to the first and second birth and another common to the third and higher-order births, and allowing

these two fertility random effects to be correlated with each other and with the mortality random effect. (The variances of these random effects and the correlations between them can only be identified in a situation where each of them affects at least two fertility transitions.) Furthermore, a situation where the contribution from the unobserved determinants is the same across transitions, except for a scale difference, can also be handled. One common fertility random effect can be included, while allowing its effect to be strengthened or weakened by transition-specific 'loadings' (with aML software). More specifically, one equation can be included for first births, one for second births, and one for third and higher-order births – each of them with the same random effect multiplied by a transition-specific 'loading'. If the 'loading' is set to 1 for one of the parity transitions it will be possible to identify the variance of the random effect, the two other 'loadings', and the correlation with the mortality random effect. (By contrast, if there are only two fertility equations, as in the present study, it is not possible to identify any 'loading').

However, reality may well be even more complex. For example, perhaps the contribution from the unobserved factors is actually specific to each parity transition – not to pairs of transitions as mentioned above – or perhaps the contribution varies with an observed determinant such as education (i.e., is involved in an interaction effect). Such patterns cannot be identified with a multilevel-multiprocess model (with existing software), and cannot or should not be assumed; simpler assumptions have to be made. It is possible that the estimates will be very biased in such situations, and it would be interesting to see examples of the size of this bias in future studies.

6.3 The challenge in brief, and a suggested strategy

To conclude, the sibling model cannot be used to separate effects of maternal age and birth year. A multilevel-multiprocess model that also controls for constant unobserved mortality determinants linked to fertility may be a reasonable alternative, but there is a potential problem with this approach: It requires explicit assumptions, not least about the underlying fertility process and the role played in it by the unobserved factors linked to the constant unobserved mortality determinants. These assumptions may accord poorly with reality, so that the estimates may differ significantly from the truth. By contrast, a sibling model requires no such assumptions about the fertility process and its link to the constant unobserved mortality determinants.

A good strategy may be to start with a sibling model without birth year included (and where the estimated effect of maternal age will reflect the sum of the effects of maternal age and birth year, net of the covariates in the model), and subsequently estimate a multilevel-multiprocess model without birth year included but otherwise

with the same variables, to the extent that they are relevant. When estimating the latter model, one should experiment with different assumptions about the random effects (within the possibilities offered by the estimation software). It would also make good sense to include as many theoretically meaningful observed determinants as possible in the fertility and (in particular) the mortality equations and allow their effects to be complex enough to reflect what is known from other sources about the effects of these determinants. It would be reassuring to see quite similar estimates from the sibling model and the multilevel-multiprocess model. Birth year can then be added to the multilevel-multiprocess model in the next step. Generally, if the intention is to separate effects of maternal age and birth order or birth interval, finely categorized variables should be used, although this is not as important with a multilevel-multiprocess model as in a sibling analysis.

If the data does not allow a multilevel-multiprocess model to be estimated, or if there is considerable doubt about the appropriateness of such a model after comparison with a corresponding sibling model such as just described, a short-cut could be considered: a birth-year effect can simply be estimated from another type of model, possibly with other data, and this estimate subtracted from the estimated effect of the maternal-age variable in a sibling analysis to get an impression of the 'pure' effect of maternal age.

Leaving aside the problem related to birth year, it is important to be aware that estimates from both sibling models and multilevel-multiprocess models such as those used here may be biased if the unobserved factors are time-varying. One example is that a long birth interval may be the result of temporary maternal health problems that also affect child mortality or another outcome. Time-varying unobserved determinants of, for example, child mortality can to some extent be taken into account in a multilevel-multiprocess analysis by allowing the mortality random effect to have 'loadings' that vary with birth order, while a corresponding step is not possible in a sibling model. In other words, even though a sibling model has the advantage that no explicit assumptions about the link between the unobserved mortality determinants and the reproductive factors are needed, the fact that these unobserved factors are assumed to be constant means that the use of a sibling model is not necessarily the preferred 'gold standard' if there is no desire to separate effects of maternal age and birth year. An additional disadvantage of the sibling model is that the analysis must be restricted to families with two or more children, or three or more if the interest lies in the effects of birth interval lengths.

7. Broadening the perspective

A linear-dependence problem such as discussed here may also arise when other age-related variables are considered – for example, when siblings who have experienced parental divorce are analysed with the intention of finding out how the children's age at the time of the divorce affects a certain outcome. The prevalence of that outcome may have generally increased or decreased over time, and one may want to separate out this birth-year effect. (The essence of the problem is then that birth year plus age at divorce is the year of divorce, which does not vary between the siblings.)

Another version of the linear-dependence problem arises in individual fixed-effects analyses; for example, when analysing how certain individual outcomes vary by time before and after certain important life events such as having a child, marrying, or losing a close family member. In such investigations, the researcher may want to take into account that the outcomes under study also depend on the individual's age (and age minus time since the event equals the age at the event, which does not vary across time).

Estimation of multilevel-multiprocess models may not always be a feasible or reasonable option in the first-mentioned situation, and is not relevant in the second, but it may help to include the sibling groups or individuals who have not experienced the event under study. Information about these people would then be used in the estimation of the effect of birth year (in the first situation) or age (in the second situation), the implicit assumption being that effects of birth year or age are the same for them as for those who have experienced the event (Kravdal and Grundy 2019; van den Berg et al. 2014). However, a careful examination of how well such a procedure actually works in different situations remains to be done.

8. Acknowledgements

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Appendix

Table A-1: **Averages of effects on infant mortality estimated from 100 simulated populations, using different categorizations of maternal age and birth year in the estimation**

	Effects used in simulation[a]	Estimated effects in sibling fixed-effects logistic models[b]				
		Size of categories (in years) for Maternal age				
		1	2	2	3	5
		Birth year				
		2	1	2	5	3
Maternal age						
17	0.66	0.92	0.10	0.45	0.48	0.15
18	0.60	0.85				
19	0.54	0.77	0.07	0.36		
20	0.48	0.69			0.34	
21	0.42	0.60	0.03	0.27		
22	0.36	0.52				0.05
23	0.30	0.44	0.00	0.18	0.19	
24	0.24	0.36				
25	0.18	0.28	−0.03	0.09		
26	0.13	0.21			0.07	
27	0.09	0.15	−0.03	0.03		0
28	0.05	0.09				
29	0.02	0.04	0	0	0	
30	0	0				
31	−0.02	−0.04	0.05	−0.01		
32	−0.04	−0.08			−0.05	0.03
33	−0.06	−0.12	0.09	−0.03		
34	−0.08	−0.16				
35	−0.08	−0.19	0.16	−0.03	−0.07	
36	−0.08	−0.20				
37	−0.08	−0.23	0.24	−0.01		0.11
38	−0.08	−0.24			−0.07	
39	−0.08	−0.26	0.34	0.02		
40	−0.08	−0.27				
41	−0.08	−0.32	0.39	0.01	−0.10	
42	−0.08	−0.35				0.16
43	−0.08	−0.34	0.48	0.03		
44	−0.08	−0.39			−0.13	
Previous interval						
1	0.57	0.57	0.58	0.58	0.59	0.60
2	0.07	0.07	0.07	0.08	0.08	0.09
3	0	0	0	0	0	0
4	−0.05	−0.05	−0.05	−0.05	−0.06	−0.06
5–6	−0.12	−0.12	−0.12	−0.13	−0.15	−0.15
7–9	−0.12	−0.12	−0.12	−0.14	−0.18	−0.18
10+	−0.12	−0.12	−0.12	−0.16	−0.25	−0.24

Table A-1:   (Continued)

	Effects used in simulation[a]	Estimated effects in sibling fixed-effects logistic models[b]				
		Size of categories (in years) for Maternal age				
		1	2	2	3	5
		Birth year				
		2	1	2	5	3
Birth order						
1	0	0	0	0	0	0
2	0.10	0.10	0.10	0.08	0.04	0.03
3	0.20	0.20	0.19	0.16	0.09	0.07
4	0.30	0.30	0.29	0.25	0.14	0.12
5	0.40	0.40	0.39	0.33	0.19	0.17
Birth year						
1967	0.26	−0.01	0.80	0.32	0.11	0.44
1968	0.24		0.74			
1969	0.22	−0.01	0.69	0.27		
1970	0.20		0.62			0.32
1971	0.18	−0.01	0.56	0.22		
1972	0.16		0.49		0.06	
1973	0.14	−0.01	0.44	0.16		0.22
1974	0.12		0.37			
1975	0.10	−0.01	0.31	0.10		
1976	0.08		0.24			0.10
1977	0.06	0.00	0.18	0.05	0	
1978	0.04		0.12			
1979	0.02	0	0.06	0		0
1980	0		0			
1981	−0.02	0.00	−0.07	−0.06		
1982	−0.04		−0.13		−0.07	−0.11
1983	−0.06	0.00	−0.20	−0.12		
1984	−0.08		−0.25			
1985	−0.10	0.00	−0.32	−0.17		−0.21
1986	−0.12		−0.38			
1987	−0.14	0.00	−0.44	−0.23	−0.12	
1988	−0.16		−0.51			−0.32
1989	−0.18	0.00	−0.57	−0.29		
1990	−0.20		−0.64			
1991	−0.22	0.00	−0.70	−0.34		−0.41
1992	−0.24		−0.76		−0.17	
1993	−0.26	0.00	−0.82	−0.39		
1994	−0.28		−0.88			−0.50
1995	−0.30	0.00	−0.94	−0.44		
1996	−0.32		−1.00			
1997	−0.34	0.00	−1.06	−0.50	−0.22	−0.58
1998	−0.36		−1.13			
1999	−0.38	0.00	−1.19	−0.54		
2000	−0.40		−1.24			−0.67
2001	−0.42	0.00	−1.32	−0.61		

Table A-1: (Continued)

	Effects used in simulation[a]	Estimated effects in sibling fixed-effects logistic models[b]				
		Size of categories (in years) for				
		Maternal age				
		1	2	2	3	5
		Birth year				
		2	1	2	5	3
2002	−0.44		−1.39		−0.25	
2003	−0.46	0.02	−1.42	−0.63		−0.71
2004	−0.48		−1.48			
2005	−0.50	0.01	−1.55	−0.69		
2006	−0.52		−1.63			−0.85
2007	−0.54	−0.08	−1.78	−0.82	−0.35	
2008	−0.56		−1.89			

Note: [a] The model used in the simulation was as explained in the text. [b] The model included the variables shown in this table plus sibling fixed effects. The estimates that are shown for age or birth year x are for a category including ages or birth years from x to x, x+1, x+2, or x+4 (varying across columns as indicated by the column heading), except that the highest age category only includes ages up to 44 and the highest category for birth year only includes birth years up to 2008.

Table A-2: Averages of effects on infant mortality estimated from 100 simulated populations, using different categorizations of maternal age and leaving birth year out of the model in the estimation (but not in the simulation)

	Effects used in simulation[a]	Estimated effects in sibling fixed-effects logistic models[b]	
		Size of categories for maternal age	
Maternal age		1 year	3 years
17	0.66	0.92	0.58
18	0.60	0.85	
19	0.54	0.77	
20	0.48	0.68	0.41
21	0.42	0.60	
22	0.36	0.52	
23	0.30	0.44	0.24
24	0.24	0.36	
25	0.18	0.28	
26	0.13	0.21	0.09
27	0.09	0.15	
28	0.05	0.09	
29	0.02	0.04	0
30	0	0	
31	−0.02	−0.04	
32	−0.04	−0.08	−0.07
33	−0.06	−0.12	
34	−0.08	−0.16	

Table A-2:   (Continued)

	Effects used in simulation[a]	Estimated effects in sibling fixed-effects logistic models[b]	
		Size of categories for maternal age	
		1 year	3 years
35	−0.08	−0.19	−0.12
36	−0.08	−0.20	
37	−0.08	−0.23	
38	−0.08	−0.24	−0.14
39	−0.08	−0.26	
40	−0.08	−0.27	
41	−0.08	−0.32	−0.19
42	−0.08	−0.34	
43	−0.08	−0.34	
44	−0.08	−0.38	−0.23
Previous interval			
1	0.57	0.57	0.60
2	0.07	0.07	0.08
3	0	0	0
4	−0.05	−0.05	−0.07
5–6	−0.12	−0.12	−0.16
7–9	−0.12	−0.12	−0.19
10+	−0.12	−0.12	−0.28
Birth order			
1	0	0	0
2	0.10	0.10	0.03
3	0.20	0.20	0.06
4	0.30	0.30	0.10
5	0.40	0.40	0.17

Note: [a] The model used in the simulation was as explained in the text. [b] The model included the variables shown in the tables plus sibling fixed effects. The estimates that are shown for age x in the third column are for a category including ages x to x+2, except that the highest age category only includes age 44.