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Abstract

Knowledge about how information is stored in neurons of animals and in the human brain is still incomplete. A hypothesis related to long-term changes in synaptic efficiency has strong experimental support, but does not seem to be able to explain all observations. It has recently been proposed that magnetite together with a prion-like protein could be involved in a tandem mechanism for storage of memory in neurons in which electric impulses are received and reshaped by the magnetite to a form which can be accepted by the protein. The magnetite crystals can be magnetized by an electrical impulse, but it cannot hold the magnetism, which drops to zero after each impulse. Therefore, magnetite cannot be the substance in which information is stored. In the present paper we explain how a tandem mechanism could function in a neuron in which magnetite is situated together with a prion-like protein close to the cell membrane of the axon. We assume in addition that the information is stored in special storage neurons. With this we propose a new hypothesis for information storage in neurons which could operate in addition to synaptic plasticity, but perhaps in different neurons.

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Corresponding Author	Lars Walloe
Order of Authors	Erik Alfsen, Fredrik Størmer, Arild Njå, Lars Walloe
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UNIVERSITY OF OSLO
FACULTY OF MEDICINE

Lars Walløe

Professor emeritus
Department of Physiology
Institute of Basic Medical Sciences
P.O.Box 1103 Blindern
0317 Oslo
Norway

Sognsvannsveien 9

Phone: +47 22 85 12 18
Fax: +47 22 85 12 49
E-mail: lars.walloe@medisin.uio.no
http://folk.uio.no/larswa/

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Editor,
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Manuscript: **A proposed tandem mechanism for memory storage in neurons involving magnetite and prions**

In our submitted manuscript, we propose a tandem mechanism for memory storage which involves magnetite and prions. The magnetization of magnetite can only be explained by a quantum mechanical process. The quantum mechanical argument is probably too detailed to be included in the article. However, if our manuscript is accepted for publication in Medical Hypotheses, we shall archive a six page note on the magnetization of magnetite chains in neurons in Cornell University Library: **arXiv.org** - in its section for Mathematical Physics. The note is written by our first author Erik M. Alfsen. I have attached this note on the magnetization of magnetite chains in neurons to this letter in case you would like to attach it as supplementary information to potential reviewers of the submitted manuscript.

On behalf of the authors,

Yours sincerely

Lars Walløe

Department of Physiology

University of Oslo

1   *Manuscript title:*
2   A proposed tandem mechanism for memory storage in neurons involving
3   magnetite and prions
4
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6   Memory storage with magnetite and prions
7
8   Erik M. Alfsen, Department of Mathematics, University of Oslo, Norway
9   Fredrik C. Størmer, Norwegian Institute of Public Health, Oslo, Norway
10 Arild Njå, Division of Physiology, Institute of Basic Medical Sciences, University
11 of Oslo, Norway
12 Lars Walløe, Division of Physiology, Institute of Basic Medical Sciences,
13 University of Oslo, Norway
14
15 Correspondence should be addressed to LW (lars.walloe@medisin.uio.no)
16
17 Submitting and Corresponding author:
18 Professor Lars Walløe MD PhD
19 Division of Physiology,
20 Institute of Basic Medical Sciences
21 University of Oslo
22 P.O.Box 1103 Blindern
23 NO-0317 Oslo
24 Norway
25
29

30   **Abstract**

31

32   Knowledge about how information is stored in neurons of animals and

33   in the human brain is still incomplete. A hypothesis related to long-term

34   changes in synaptic efficiency has strong experimental support, but does not

35   seem to be able to explain all observations. It has recently been proposed that

36   magnetite together with a prion-like protein could be involved in a tandem

37   mechanism for storage of memory in neurons in which electric impulses are

38   received and reshaped by the magnetite to a form which can be accepted by

39   the protein. The magnetite crystals can be magnetized by an electrical

40   impulse, but it cannot hold the magnetism, which drops to zero after each

41   impulse. Therefore, magnetite cannot be the substance in which information

42   is stored. In the present paper we explain how a tandem mechanism could

43   function in a neuron in which magnetite is situated together with a prion-like

44   protein close to the cell membrane of the axon. We assume in addition that

45   the information is stored in special *storage neurons.*   With this we propose a

46   new hypothesis for information storage in neurons which could operate in

47   addition to synaptic plasticity, but perhaps in different neurons.

48

49   **Keywords:** memory storage, magnetite, prions

50

Introduction

52

53   Knowledge of the mechanisms for storage of memory in neurons is

54   incomplete. A hypothesis related to long-term changes in synaptic strength

55   and the growth of new synaptic connections has strong experimental support

56   (Kandel, 2001).   In addition, nonsynaptic processes, such as changes in

57   neuronal excitability, may contribute to some forms of learning and memory

58   (Mozzachiodi and Byrne, 2010).   The formation of long-term memories

59   requires protein synthesis, and the gene regulation involved in such processes

60   has been shown to depend on special protein kinases (Giese and Mizuno,

61   2013) and more recently, on prion-like proteins (Si and Kandel, 2016) .

62   Prion-like proteins (here referred to as "prions" for short) have the

63   crucial property that they can change shape from a normal resting form to an

64   alternate chain-formed version.   For brevity, we refer to a prion in the

65   normal form as being in the "ground state", and to one in the alternate form

66   as being in the "excited state". In Si and Kandel (2016) it is explained how a

67   prion can be switched from the ground state to the excited state by an

68   intricate *prion-chain reaction* triggered by an electric impulse.

69	It has also been proposed that magnetite is involved in the storage of

70	information in neurons (Størmer and Laane, 2009, and Banaclocha et al,

71	2010). Magnetite is an iron oxide (Fe_3O_4) which is widely found in living

72	organisms without being involved in any known biochemical reactions.

73	Electron microscopy has indicated the presence of a minimum of 5

74	million single-domain magnetite crystals per gram in human brain tissue.

75	Black strings of aggregated particles can be extracted from brain tissue and

76	can be viewed under low power through an optical dissecting microscope. The

77	crystals range in size between 10 and 70 nm in diameter (Kirschvink et al,

78	1992). Magnetite crystals have also been found in bacteria and fish, where

79	they were seen to be organized in membrane-bound chains with up to 80

80	single-domain crystals per chain (Kirschvink et al, 1985, 2001). Such chains

81	appear to be present across all animal species.

82	The particles in these chains are formed of magnetite crystals that are

83	different from those formed through geological processes. The crystal

84	morphology of magnetite in living organisms, referred to as *nanocrystalline*

85	*magnetite*, is in fact particularly well suited for information storage for various

86	reasons. Here, we only wish to point out that nanocrystalline magnetite has

87	the crucial property that it can easily be magnetized by electrical impulses, but

88	cannot hold the magnetism, which drops to zero after each impulse.

89   Therefore, nanocrystalline magnetite cannot be the substance in which

90   information is stored. But it has recently been suggested that magnetite

91   chains together with prions could be involved in a tandem mechanism in

92   which incident electric impulses are received and reshaped by the magnetite,

93   giving it a form that can be accepted by prion-like proteins in which the

94   information can be permanently stored (Størmer and Alfsen, 2015).

95   In this paper we will explain how such a mechanism may function in a

96   neuron where a nanocrystalline magnetite chain is located together with a

97   prion close to the cell membrane, and how the electric impulses could trigger

98   prion-chain reactions such that the information is stored in the prion.

99

100

101   **A storage cell hypothesis**

102

103   Nerve impulses (action potentials) are all-or-none electrical signals

104   that encode neural information for long-distance communication along

105   nerve fibers (axons). Action potentials have large, fixed amplitudes (about

106   100 mV), short durations (about 1 ms) and travel through long axons at

107   speeds that range from below 1 to above 100 m s^{-1}, depending on the axon

108   diameter and degree of myelination.

109   Action potentials are elicited if the membrane potential is depolarized

110   to a threshold level. The threshold is lowest in the initial segment of the axon.

111   After an action potential is generated at this site, it can spread without decay

112   down the axon and all of its branches, and may (although with signal decay)

113   also invade the cell body and proximal dendrites. Graded transmembrane

114   potentials show exponential decay with distance from the generation site in

115   the neuron, but multiple small potentials may summate so that the threshold

116   is reached and an action potential is fired in the initial segment of the axon,

117   even if each input is too small to reach the threshold alone.

118   The transmembrane ionic currents associated with the passage of a

119   propagated action potential (an inward Na^+ current through voltage-gated

120   sodium channels, and, after a brief delay, an outward K^+ current through

121   voltage-gated potassium channels) will generate a local rotating current that

122   could potentially magnetize a properly oriented rod of magnetite

123   nanocrystals, as we explain here. We propose a tandem mechanism by which

124   the propagation of an action potential in an axon could, given that specific

125   conditions are satisfied, magnetize a local rod of magnetite nanocrystals that

126   in turn could change the conformation of a local prion and store a permanent

127   memory of the event.

128

129 *How can electrical signals magnetize the magnetite chain?*

130 According to the tandem hypothesis, information is stored in neurons

131 (here referred to as *storage neurons*) which contain a magnetite chain

132 together with a prion that can be in one of two possible states. More

133 specifically, each storage neuron is assumed to contain just one magnetite

134 chain, and one bit of information is stored in the neuron when the prion is

135 switched from the ground state to the excited state.

136 In a storage neuron, the bursts of Na^+ ions into the cell and K^+ ions out

137 of the cell create vortices that rotate on their axes like short-lived electrical

138 currents. When such vortices rotate around a magnetite chain, they can

139 magnetize it. These vortices arise when two oppositely directed flows of ions

140 pass through neighboring membrane areas. If P1 and P2 are sufficiently close

141 ion channels, the two flows will interact and produce vortices that rotate

142 clockwise in the (*x-z*)-plane (Figure 1).

143 According to this argument, incident signals in a storage neuron can

144 create vortices of short-lasting electrical currents that rotate like a whirlwind

145 around the magnetite chain, and these currents can magnetize the chain.

146 However, the magnetite chain differs from an electromagnet, as described

147 below.

148

149   *How can the magnetite chain change the shape of the prion?*

150   We have seen that the magnetite chain in a storage neuron can be

151   magnetized by currents rotating around it, but it is not so easy to explain how

152   the magnetic chain can change the shape of the prion, since it requires an

153   electric and not a magnetic impulse to do this. Here it may be helpful to use

154   an analogy with a familiar electric device, a transformer, which sets up a

155   pulsating electromagnetic field that produces eddy currents.

156   In the same way, when the magnetite chain of a storage neuron is

157   magnetized by electric pulses, it will set up a pulsating electric field which

158   produces ionic currents. These will rotate in right-handed vortices around the

159   magnetized magnetite chain like a whirlwind. A detailed proof of this can be

160   obtained by an elementary argument, but all we need to know here is that the

161   electric field at a given point is largest close to the magnetized chain, and that

162   it decreases rapidly with increasing distance from the chain.

163   It is thus conceivable that a magnetite chain can indeed act like a pulse

164   transformer, and "assemble" and "coordinate" many small electric currents

165   around a magnetite chain which is located together with a prion close to the

166   cell membrane, and that it can replace these currents by a strong current

167   which rotates in the same way around the magnetite chain and is strong

168   enough to magnetize it.

169   But this needs a more detailed explanation. For this, we need to know

170   more about the magnetic properties of the magnetite chains, which are quite

171   different from the magnetic properties of iron cores in standard transformers.

172

173   *Why is nanocrystalline magnetite particularly suited for the storage of*

174   *information in neurons?*

175   Magnetite (Fe_3O_4) is an iron oxide which differs from ordinary rust (Fe_2

176   O_3)   in its crystalline structure and its specific magnetic properties. In

177   molecules of ordinary rust, all the innermost electronic shells are filled, but in

178   molecules of magnetite, there is one loosely bound electron outside the full

179   innermost shells. The extraordinary magnetic properties of magnetite are

180   largely due to this electron. The crystal structure of nanocrystalline magnetite

181   in neurons differs from that of magnetite in geological deposits. Geological

182   magnetite crystals are usually octahedral, whereas those of biological

183   magnetite are cubo-octahedral. The individual crystals in a magnetite chain

184   are tiny magnets which are aligned "north pole to south pole"   along the axis

185   of the chain, each with its poles in the faces of the crystal lying perpendicular

186   to the axis.

187   We know that magnetization of ferromagnetic substances involves

188   crystal *domains*, each usually containing a large number of crystals, and that

189   the magnetization process consists of a rotational displacement of these

190   domains so that they are oriented in the direction of the magnetizing

191   magnetic field (Harnwell (1949)).

192   The magnetite chain in a storage neuron cannot be magnetized by

193   rotation of its single crystal domains since they are tiny magnets and are

194   already oriented north pole to south pole along the chain axis. However, they

195   can be magnetized by a quantum mechanical process. This is based on

196   quantum mechanical resonance between a strong magnetizing exterior field

197   and the weak magnetic fields of the crystals in the chain, which are forced to

198   oscillate in the same quantum mechanical phase as the magnetizing field. The

199   process is explained in more detail in Alfsen (2018) where the phases in

200   question are related to the complex wave functions of quantum mechanics.

201   Here we explain the idea in non-mathematical terms. For this purpose we will

202   make use of the old quantum theory that the electrons in an atom rotate in

203   orbits around the atomic nucleus. In this picture, the loosely bound electron in

204   the magnetite molecule may be thought of as a particle rotating in orbit

205   around a 'core' consisting of the atomic nuclei of iron and oxygen and the

206   other tightly bound electrons.

207   When a storage neuron is in the resting state, the crystals in its

208   magnetite chain are completely unrelated in phase. But when the chain is

209 subject to a sufficiently strong exterior magnetic field, all crystals in the chain

210 are forced to oscillate in the same phase as the exterior field. After this, the

211 chain can no longer contain crystals in opposite phase whose magnetic fields

212 erase each other out. Therefore, the crystals are now all in the same phase as

213 the magnetizing exterior field, and the chain is magnetized up to its greatest

214 possible strength.

215   The magnetization of a magnetite chain by this process requires a

216 certain critical energy level. If the critical energy level is exceeded, the chain

217 will always be magnetized up to the same maximal strength. The critical

218 energy level is here called the *magnetization energy* of the chain. Thus, no

219 magnetization of the magnetite chain of a storage neuron will occur if it is

220 subjected to energy levels below the magnetization energy, whereas any

221 energy level above this magnetization energy will magnetize it up to its full

222 strength but no further, however strong the signal is. This means that the

223 magnetization process of the magnetite chain in a storage neuron is different

224 from that of the iron core in an electromagnet, in which the magnetism

225 increases proportionally with the strength of the magnetizing current.

226   Each domain in the nanocrystalline magnetite in neurons contains a

227 single crystal. These crystals are of the same shape and size, and they also act

228 as tiny magnets. When the magnetic chain of a storage neuron is magnetized

229 up to its maximal strength, the magnetic energy of the chain is equal to the

230 magnetic energy of a single crystal multiplied by the number of crystals in the

231 chain. The magnetization energy of a magnetite chain therefore depends only

232 on its length, i.e. on the number of crystals in the chain.

233   According to the tandem hypothesis, one bit of information is stored

234 when the prion in a storage neuron is switched from the ground state to the

235 excited state. This requires a certain amount of energy which must be

236 provided by the magnetized magnetite chain. We call this energy the

237 *excitation energy* of the prion.

238   In order to raise the prion from the ground state to the excited state,

239 the incident signal must provide sufficient energy to magnetize the chain, and

240 the chain must in turn provide enough energy to create "electrical

241 whirlwinds" that can excite the prion. This means that the magnetization

242 energy held by the magnetite chain, which depends on its length, must exceed

243 the excitation energy of the prion. The necessary amount of energy can

244 therefore only be provided by a magnetite chain of a certain length. The

245 minimum chain length needed for excitation of the prion is here called the

246 *critical chain length* for the magnetic chain in a given storage neuron.

247   The discussion above shows that there are two conditions which must

248 be satisfied by a storage neuron in a pathway. The first pertains to the storage

249   neuron itself, which must be able to provide the magnetization energy needed

250   to magnetize the magnetite chain to its maximum strength, and the second

251   pertains to the magnetite chain, which must contain enough crystals to be

252   above the critical chain length.

253   The quantum mechanical process for magnetization of nanocrystalline

254   magnetite requires much less energy than the magnetization of geological

255   magnetite, which involves physical displacement of large domains. This

256   explains why nanocrystalline magnetite has the crucial property that it is easy

257   to magnetize but cannot hold the magnetism.

258

259

260   **Discussion**

261

262   How can the proposed tandem mechanism function in neurons?

263   White-Grindley et al. (2014) point out that: "- - - a central question is

264   how the conversion of neuronal CPEB /cytoplasmic polyadenylation element

265   binding protein/ to the amyloidogenic state is regulated to confer activity

266   dependence and restrict it to the relevant neuron/ synapse." In our tandem

267   hypothesis context, this is the question of how the process that changes the

268 state of a prion is regulated to take place at just the right time and the right

269 place in the relevant neuron or synapse.

270   It has been an open question whether memory storage takes place only

271 in synapses or also in specialized neurons. We assume that both entities may

272 be involved in the storage process. If information storage is based on the

273 summation of a large number of local signals, the important entities are the

274 synapses at which the incident signals are targeted. But this is not the case for

275 information storage in storage neurons based on the proposed tandem

276 mechanism. In this case, the most important entities are not the synapses but

277 the initial segments of the axons of the post-synaptic neurons where action

278 potentials are initiated and where magnetite chains are presumed to be

279 located.

280   Here we discuss the propagating action potentials in a pathway of axons

281 which terminates in a storage neuron. The magnetite chain in a storage

282 neuron can be magnetized most efficiently if it is located laterally and near

283 the cell membrane. We say that the storage neuron has a "threshold density

284 of ion channels" if there are enough Na^+ and K^+ channels near the magnetite

285 chain to release the rotating currents of ions that can magnetize the chain.

286   We argue, based on the quantum mechanical properties of

287 nanocrystalline magnetite, that the *magnetization energy* of the magnetite

288   chain depends only on the length of the chain, which must be above the

289   *critical length* to provide the *excitation energy* needed to switch the prion in

290   the cell from the ground state to the excited state.

291   The following three assumptions are required as part of the tandem

292   hypothesis for memory storage by magnetite and prions:

293   A1. The magnetite chain in a storage neuron is located close to the cell

294   membrane and is laterally oriented in the neuron.

295   A2. The density of ion channels in the storage neuron is above the

296   threshold level.

297   A3. The length of the magnetite chain in the storage neuron is above

298   the critical length.

299   Given these assumptions, the tandem hypothesis explains how a

300   propagating signal can magnetize the magnetite chain in a storage neuron,

301   which in turn raises the prion in the storage neuron to the excited state and

302   thereby stores one bit of information.

303   The proposed tandem mechanism converts electrical signals in the form

304   of action potentials and propagated along axon pathways to a form that can

305   be accepted by special "storage neurons", each containing a chain of

306   nanocrystalline magnetite and a prion which can be switched from a "ground

307   state" to an "excited (amyloidogenic) state" when the chain is magnetized by

308   incident signals. This process stores one bit of information in the cell.

309

310   *Conclusions*

311   According to the tandem hypothesis, the storage neurons must satisfy

312   three assumptions. The first of these specifies the location of the magnetite

313   chain in the cell, the second requires a sufficient number of Na^+ and K^+ ion

314   channels near the magnetite chain to permit the ion bursts that magnetize the

315   chain, and the third requires the magnetite chain to be long enough to

316   provide sufficient energy to raise the prion to the excited state and store

317   memory in the cell

318   If cells are to function as storage neurons, it must also be possible to

319   retrieve stored information when needed, through processes that are

320   dependent on whether the prion is in the excited state or the ground state. It

321   is conceivable that such retrieval of information could be achieved by sending

322   a test signal into the cell via an appropriate synaptic connection.

323

324

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369

370

371

372

373 Figure 1

374

375 Two oppositely directed bursts of ions through neighboring ion channels can

376 create vortices that magnetize the magnetite chain in a storage neutron.

377   The action potential travels along the cell membrane in the x-direction.

378 Near point P_2 the sodium channels are open and near the point P_1 the

379 potassium channels. The magnetite chain S-N is posed laterally close to the

380 cell membrane and oriented in the y-direction.

381

He authors report no conflict of interest

A note on magnetization of magnetite chains in neurons

Erik M. Alfsen

February 3, 2018

In article [1], it is proposed that information can be stored in a neuron when a magnetized chain of magnetite crystals is reshaped and amplified by electric impulses that are switching a prion from its ground state to its excited state. In this article, this process is explained without use of mathematics. But in the present note, we will present the quantum mechanical equations for the process in which such a chain of crystals is magnetized for this purpose.

We know that quantal quantities are denoted by complex-valued functions $\psi = re^{i\varphi}$ defined on 4-dimensional space-time (or more general real spaces not needed here), and that the absolute value $|\psi|$ gives the probability density of the result of measurements of such quantities. We also know that quantum mechanical superposition of two quantal quantities is represented by the sum of their complex wave functions. Actually, each quantum mechanical wave function ψ consists of two spin-components, here denoted by ψ_+ and ψ_-. The mathematical theory of spin dates back to Pauli's 1927-article: On the quantum mechanics of the magnetic electron [2]. Here we will give a brief survey, based on the presentation in the book [3], of those parts of Pauli's theory on electron spin which will be needed in the present note.

From the two spin-components ψ_+ and ψ_- we can construct a two-dimensional vector $\boldsymbol{\Psi}$ (not in physical space but in two-dimensional complex space) and the vector conjugate to it

$$\boldsymbol{\Psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \qquad \boldsymbol{\Psi}^\dagger = (\psi_+^*, \psi_-^*) \ . \tag{1}$$

As explained in [2; Sec. 10.1], the magnetic moment-vector of Pauli's magnetic electron is given by an equation of the form

$$\boldsymbol{\mu} = \mu \boldsymbol{\Sigma} \ , \tag{2}$$

1

where μ is a constant (the scalar magnetic moment), and $\boldsymbol{\Sigma}$ is a three-dimensional vector (in physical space) given by the following equation for the coordinates

$$\Sigma_i = \boldsymbol{\Psi}^\dagger \sigma_i \boldsymbol{\Psi}, \qquad i = 1, 2, 3 \quad \text{or} \quad i = x, y, z, \tag{3}$$

where σ_i are the *Pauli matrices*

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

(Here (1) is eq. (10.16) and (3) is eq. (10.12) in [3].)

Multiplying out the matrix product in (3) gives the following equations (which are (10.6a), (10.6b), (10.6c) in [3])

$$\Sigma_x = \psi_+^* \psi_- + \psi_-^* \psi_+ \tag{5a}$$

$$\Sigma_y = -i(\psi_+^* \psi_- - \psi_-^* \psi_+) \tag{5b}$$

$$\Sigma_z = \psi_+^* \psi_+ - \psi_-^* \psi_- . \tag{5c}$$

The spin-components of Pauli's magnetic electron are supposed to transform as follows under rotation by angle α around a straight line in space

$$'\psi_+ = e^{-i\alpha/2}\,\psi_+ , \qquad '\psi_- = e^{i\alpha/2}\,\psi_- . \tag{6}$$

By these equations, the wave function changes sign after rotation by the angle $\alpha = 2\pi$. But this won't cause problems, as it is not the wave function itself that has physical meaning, but quantities constructed from it, which are invariant under a sign-change.

The key example is the squared norm of the associated two-dimensional vector

$$|\boldsymbol{\Psi}|^2 = (\psi_+^*, \psi_-^*) \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = |\psi_+|^2 + |\psi_-|^2 , \tag{7}$$

which is interpreted as the probability density of finding the electron with given spin-state at a given point in space-time [3; eq. (10.5)]. What we are here interested in, is the transformation of the magnetic moment-vector of an electron whose wave-function is transformed as in (6).

We know that the components of the vector $\boldsymbol{\Sigma}$, which determines the magnetic moment-vector of the electron, is given as in (5). From this, we first observe that the z-component of $\boldsymbol{\Sigma}$ is transformed as follows

$$'\Sigma_z = \psi_+^* \psi_- - \psi_-^* \psi_+ = \Sigma_z . \tag{8}$$

2

Thus, the z-component of $\boldsymbol{\Sigma}$ remains unchanged under this transformation of the wave function. This fact is of course no mere coincidence: the transformation formula (6) was tailored to achieve just this!

To determine the transformation of Σ_x and Σ_y, we assume that the wave function of the electron at a given point (x, y, z) at a given time, say $t = 0$, is given by the equation

$$\psi_+ = r_+ \, e^{i\varphi_+} \, , \qquad \psi_- = r_- \, e^{i\varphi_-} \, , \tag{9}$$

with the *amplitudes* r_+, r_- and the *phase angles* φ_+ and φ_- .

Note that the two terms $\psi_+^* \psi_-$ and $\psi_-^* \psi_+$ occurring in (5a) and (5b) are given as follows in terms of the *phase difference* $\varphi = \varphi_+ - \varphi_-$

$$\psi_+^* \psi_- = r_+ r_- \, e^{-i\varphi}, \qquad \psi_-^* \psi_+ = r_+ r_- \, e^{i\varphi},$$

and that the two terms $\psi_+^* \psi_+$ and $\psi_-^* \psi_-$ occurring in (5c) are given by

$$\psi_+^* \psi_+ = r_+^2 \, , \qquad \psi_-^* \psi_- = r_-^2 \, .$$

By (6) the spin-components of an electron transform as follows under rotation by the angle α

$$'\psi_+ = r_+ e^{i(\varphi_+ - \alpha/2)} \, , \qquad '\psi_+ = r_+ e^{i(\varphi_- - \alpha/2)} \, .$$

From this it follows that the two terms $\psi_+^* \psi_-$ and $\psi_-^* \psi_+$ transform as follows

$$'(\psi_+^* \psi_-) = r_+ r_- e^{i(\varphi - \alpha)} \, , \qquad '(\psi_-^* \psi_+) = r_+ r_- e^{-i(\varphi - \alpha)} \, ,$$

and by (5 abc) the components of $\boldsymbol{\Sigma}$ transform as follows

$$'\Sigma_x = r_+ r_- \big(e^{-i(\varphi - \alpha)} + e^{i(\varphi - \alpha)}\big)$$
$$'\Sigma_y = r_+ r_- \big(e^{-i(\varphi - \alpha)} - e^{i(\varphi - \alpha)}\big)$$
$$'\Sigma_z = r_+^2 - r_-^2 \, ,$$

and from these equations we conclude that

$$'\Sigma_x = 2 r_+ r_- \cos(\varphi - \alpha) \tag{10a}$$
$$'\Sigma_y = 2 r_+ r_- \sin(\varphi - \alpha) \tag{10b}$$
$$'\Sigma_z = r_+^2 - r_-^2 \, . \tag{10c}$$

With this we have shown that when the wave function of the electron is transformed by a rotation with an angle α, such that the two-dimensional

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complex vector $\mathbf{\Psi}$ of its spin-components are transformed by means of the angle $\alpha/2$ as in (6), then the vector $\mathbf{\Sigma}$ is rotated clockwise around the z-axis by the angle α to the position given by eqs. (10 abc).

By the tandem hypothesis announced in [1], information is stored in special *storage neurons*. By definition, each storage neuron contains one and just one magnetite chain together with a prion which is switched from the *ground state* to the *excited state*, and thereby stores one bit of information when it is subject to a sufficiently strong electric impulse.

What is of interest in our present context, is the moment-vector of the magnetite chain in a storage neuron. This chain is assumed to be located close to the cell membrane in the direction of the y-axis of the chosen coordinate system, which was referred to as the "lateral direction" in [1; Sec. 6]. But in our present context it is better to use another coordinate system which is better adapted to the transformation (6) in Pauli's theory. Therefore we will now change our coordinate system by a cyclic left-shift by which the old axis (in the forward direction of the nerve signals) is the new z-axis, while the old y-axis is the new x-axis, and the old x-axis is the new y-axis. With this change of coordinate system we get a picture different from that in Fig.1 of [1; Sec. 3], as the magnetite chain is in the direction of the x-axis, not the y-axis.

In [1; Sec. 3-4] it is explained how electrical signals in a storage neuron can trigger a boost of Na^+ and K^+-ions from ion channels, which produce vortices that are rotating like blasts of a whirlwind around the magnetite chain of the neuron. These vortices initiate the quantum mechanical process which is magnetizing the chain. In this process, these positively charged ions are in rotational motion with rotation axis in the direction of the x-axis of the new coordinate system. We will assume that they are setting up a magnetic field in this direction and that the associated $\mathbf{\Sigma}$-vector is ruled by equations like those in (10 abc).

More specifically, we assume that this field is the sum of contributions from the crystals in the chain, and that each of those is given (as in (2)) by a $\mathbf{\Sigma}$-vector with x-component of the form

$$\Sigma_x = R\cos(\varphi - \alpha) \ , \qquad (11)$$

with amplitude $R > 0$ and phase angle $\cos(\varphi - \alpha)$ where $\varphi \in [0, 2\pi]$ and $\alpha = \omega t$ where the rotational velocity ω is the same for all crystals in the chain.

We will now consider a magnetite chain with n crystals, and we will denote their phase differences by $\varphi_1, \ldots, \varphi_n$ and their amplitude factors by

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$R_1, \ldots, R_n$. With this, the magnetite chain in a storage neuron can be magnetized to a linear magnet in the x-direction of the chosen coordinate system, whose magnetic moment is given by the following x-component of its complex $\mathbf{\Sigma}$-vector

$$\Sigma_x = \sum_{i=1}^{n} R_i \cos(\varphi_i - \alpha) \ . \tag{12}$$

When a storage neuron is in the resting state, then the crystals in the magnetite chain are completely unrelated in phase, so the phase differences $\varphi_1, \ldots, \varphi_n$ are randomly distributed and the two partial sums of positive and negative terms in (12) are likely to cancel each other out.

But when the storage neuron is subject to an electric impulse which magnetizes the chain as explained in [1; Sec. 3], then the crystals will no longer be unrelated in phase, as they are subject to a strong external field which is superimposed on the weak magnetic fields of the crystals. In this process the crystals will be forced to oscillate with increased amplitude in the same phase as the wave function of the external field, which is a quantal entity with its own amplitude and phase difference between its two spin-components.

This explains how the entire magnetite chain of a storage neuron may be magnetized by an electric impulse up to a strength which for a chain with sufficiently many crystals is large enough to switch the prion from its ground state to its excited state and thereby store one bit of information in the cell.

For especially interested readers, we will explain how equations in this note can be obtained from the general theory of quantum systems defined in abstract spaces of higher dimension than the 4-dimensional Minkowski-space.

In the formalism of quantum mechanics, a quantum system is represented on a complex Hilbert space, and the probability density of measurements of a quantum mechanical observable is represented by a positive operator of norm one, which is given by a positive matrix of trace one. If the range space of this representing operator is a single complex line, then it is the orthogonal projection onto this line. Observables for which this is the case, are said to be *in pure state*. A key example is the projection onto the line of the two-dimensional complex vector $\mathbf{\Psi}$ of the electron.

This complex vector $\mathbf{\Psi}$ is said to be a *spinor*, or more specifically a "two-dimensional spinor" since this is a special case of the general spinor concept. From general spinors we can construct, not only the rank one

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matrices which determine observables in pure state, but also matrices of general observables.

The matrix of each general observable is a convex combination of matrices of observables in pure state. The matrices of observables is a convex set, and the extreme points of this convex set are the matrices of observables in pure state. For simplicity, we will identify the observables with their representing matrices. With this we will use the single word "state" to denote the matrix of a general observable and the term "pure state" to denote a the matrix of an observable in pure state. We will also use the term *state space* to denote the convex set of all matrices which represent states of the given quantum system.

The state space of a quantum system is an important concept because it determines the probability density of measurements of the observables, which is given by a "quadratic form" on the representing matrices, as in the special case of the electron, for which the position is an observable in pure state with probability density given by the matrix equation (7).

Each pair of extreme points in the state space are separated by an *angular distance* introduced in Definition 4.9 in the book [4]. These two points are pure states whose matrices are projections onto lines determined by vectors in the Hilbert space, and it can be shown that the angle between these lines (defined by the inner product of the Hilbert space) is precisely twice the angular distance between the two extreme points [4: Prop.4.14].

In this book, this result is presented in a more general context, but the proof is a simple geometric argument. What is shown, is that when a vector in the Hilbert space is rotated by the angle α around the origin of the Hilbert space, then the corresponding extreme points of the state space are rotating by the double angle 2α around the state represented by the unit matrix. With this it is shown that the occurrence of the half-angles in equation (6) is a special case of a theorem valid for general quantum systems.

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[1] Erik M. Alfsen, Fredrik C. Størmer, Arild Njå, Lars Walløe. A proposed tandem mechanism for the storage of memory in neurons.

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A note on magnetization of magnetite chains in neurons

Erik M. Alfsen

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In article [1], it is proposed that information can be stored in a neuron when a magnetized chain of magnetite crystals is reshaped and amplified by electric impulses that are switching a prion from its ground state to its excited state. In this article, this process is explained without use of mathematics. But in the present note, we will present the quantum mechanical equations for the process in which such a chain of crystals is magnetized for this purpose.

We know that quantal quantities are denoted by complex-valued functions $\psi = re^{i\varphi}$ defined on 4-dimensional space-time (or more general real spaces not needed here), and that the absolute value $|\psi|$ gives the probability density of the result of measurements of such quantities. We also know that quantum mechanical superposition of two quantal quantities is represented by the sum of their complex wave functions. Actually, each quantum mechanical wave function ψ consists of two spin-components, here denoted by ψ_+ and ψ_-. The mathematical theory of spin dates back to Pauli's 1927-article: On the quantum mechanics of the magnetic electron [2]. Here we will give a brief survey, based on the presentation in the book [3], of those parts of Pauli's theory on electron spin which will be needed in the present note.

From the two spin-components ψ_+ and ψ_- we can construct a two-dimensional vector $\boldsymbol{\Psi}$ (not in physical space but in two-dimensional complex space) and the vector conjugate to it

$$\boldsymbol{\Psi} = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \qquad \boldsymbol{\Psi}^\dagger = (\psi_+^*, \psi_-^*) \ . \tag{1}$$

As explained in [2; Sec. 10.1], the magnetic moment-vector of Pauli's magnetic electron is given by an equation of the form

$$\boldsymbol{\mu} = \mu \boldsymbol{\Sigma} \ , \tag{2}$$

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where μ is a constant (the scalar magnetic moment), and $\boldsymbol{\Sigma}$ is a three-dimensional vector (in physical space) given by the following equation for the coordinates

$$\Sigma_i = \boldsymbol{\Psi}^\dagger \sigma_i \boldsymbol{\Psi}, \qquad i = 1, 2, 3 \quad \text{or} \quad i = x, y, z, \tag{3}$$

where σ_i are the *Pauli matrices*

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

(Here (1) is eq. (10.16) and (3) is eq. (10.12) in [3].)

Multiplying out the matrix product in (3) gives the following equations (which are (10.6a), (10.6b), (10.6c) in [3])

$$\Sigma_x = \psi_+^* \psi_- + \psi_-^* \psi_+ \tag{5a}$$

$$\Sigma_y = -i(\psi_+^* \psi_- - \psi_-^* \psi_+) \tag{5b}$$

$$\Sigma_z = \psi_+^* \psi_+ - \psi_-^* \psi_- . \tag{5c}$$

The spin-components of Pauli's magnetic electron are supposed to transform as follows under rotation by angle α around a straight line in space

$$'\psi_+ = e^{-i\alpha/2} \, \psi_+ \,, \qquad '\psi_- = e^{i\alpha/2} \, \psi_- \, . \tag{6}$$

By these equations, the wave function changes sign after rotation by the angle $\alpha = 2\pi$. But this won't cause problems, as it is not the wave function itself that has physical meaning, but quantities constructed from it, which are invariant under a sign-change.

The key example is the squared norm of the associated two-dimensional vector

$$|\boldsymbol{\Psi}|^2 = (\psi_+^*, \psi_-^*) \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = |\psi_+|^2 + |\psi_-|^2 \,, \tag{7}$$

which is interpreted as the probability density of finding the electron with given spin-state at a given point in space-time [3; eq. (10.5)]. What we are here interested in, is the transformation of the magnetic moment-vector of an electron whose wave-function is transformed as in (6).

We know that the components of the vector $\boldsymbol{\Sigma}$, which determines the magnetic moment-vector of the electron, is given as in (5). From this, we first observe that the z-component of $\boldsymbol{\Sigma}$ is transformed as follows

$$'\Sigma_z = \psi_+^* \psi_- - \psi_-^* \psi_+ = \Sigma_z . \tag{8}$$

Thus, the z-component of $\boldsymbol{\Sigma}$ remains unchanged under this transformation of the wave function. This fact is of course no mere coincidence: the transformation formula (6) was tailored to achieve just this!

To determine the transformation of Σ_x and Σ_y, we assume that the wave function of the electron at a given point (x, y, z) at a given time, say $t = 0$, is given by the equation

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with the *amplitudes* r_+, r_- and the *phase angles* φ_+ and φ_- .

Note that the two terms $\psi_+^* \psi_-$ and $\psi_-^* \psi_+$ occurring in (5a) and (5b) are given as follows in terms of the *phase difference* $\varphi = \varphi_+ - \varphi_-$

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and from these equations we conclude that

$$'\Sigma_x = 2 r_+ r_- \cos(\varphi - \alpha) \tag{10a}$$
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With this we have shown that when the wave function of the electron is transformed by a rotation with an angle α, such that the two-dimensional

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complex vector $\boldsymbol{\Psi}$ of its spin-components are transformed by means of the angle $\alpha/2$ as in (6), then the vector $\boldsymbol{\Sigma}$ is rotated clockwise around the z-axis by the angle α to the position given by eqs. (10 abc).

By the tandem hypothesis introduced in [1] (and anticipated in [5]), the information is stored in special *storage neurons*. By definition, each storage neuron contains one and just one magnetite chain together with a prion which is switched from the *ground state* to the *excited state*, and thereby stores one bit of information when it is subject to a sufficiently strong electric impulse.

What is of interest in our present context, is the moment-vector of the magnetite chain in a storage neuron. This chain is assumed to be located close to the cell membrane in the direction of the y-axis of the chosen coordinate system, which was referred to as the "lateral direction" in [1; Sec. 6]. But in our present context it is better to use another coordinate system which is better adapted to the transformation (6) in Pauli's theory. Therefore we will now change our coordinate system by a cyclic left-shift by which the old x-axis (in the forward direction of the nerve signals) is the new z-axis, while the old y-axis is the new x-axis, and the old z-axis is the new y-axis. With this change of coordinate system we get a picture different from that in Fig.1 of [1; Sec. 3], as the magnetite chain is in the direction of the x-axis, not the y-axis. (Fig.1 of this note).

In [1; Sec. 3-4] it is explained how electrical signals in a storage neuron can trigger a boost of Na^+ and K^+-ions from ion channels, which produce vortices that are rotating like blasts of a whirlwind around the magnetite chain of the neuron. These vortices initiate the quantum mechanical process which is magnetizing the chain. In this process, these positively charged ions are in rotational motion with rotation axis in the direction of the x-axis of the new coordinate system. We will assume that they are setting up a magnetic field in this direction and that the associated $\boldsymbol{\Sigma}$-vector is ruled by equations like those in (10 abc).

More specifically, we assume that this field is the sum of contributions from the crystals in the chain, and that each of those is given (as in (2)) by a $\boldsymbol{\Sigma}$-vector with x-component of the form

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with amplitude $R > 0$ and phase angle $\varphi - \alpha$ where $\varphi \in [0, 2\pi]$ and $\alpha = \omega t$ where the rotational velocity ω is the same for all crystals in the chain.

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The matrix of each general observable is a convex combination of matrices of observables in pure state. The matrices of observables is a convex set, and the extreme points of this convex set are the matrices of observables in pure state. For simplicity, we will identify the observables with their representing matrices. With this we will use the single word "state" to denote the matrix of a general observable and the term "pure state" to denote a the matrix of an observable in pure state. We will also use the term *state space* to denote the convex set of all matrices which represent states of the given quantum system.

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In this book, this result is presented in a more general context, but the proof is a simple geometric argument: By [4; Cor.4.8] the face generated by two distinct extreme points of the state space is a Eucledean 3-ball (say of radius r). By [4; Prop.4.11] and [4; Def.4.12] the angular distance of the two extreme points is the angle between them when viewed from the center of the ball. But the angle between them is smaller when viewed from the origin of the Hilbert space, as this point is at a longer distance ($\sqrt{2}\, r$) from the two extreme points.

From this it follows that the angular distance between the two given extreme points (pure states) is precisely twice the angle between their representing vectors in the Hilbert space [4; Prop.14]. With this it is shown that the occurrence of the half-angles in equation (6) is a special case of a theorem valid for general quantum systems.

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